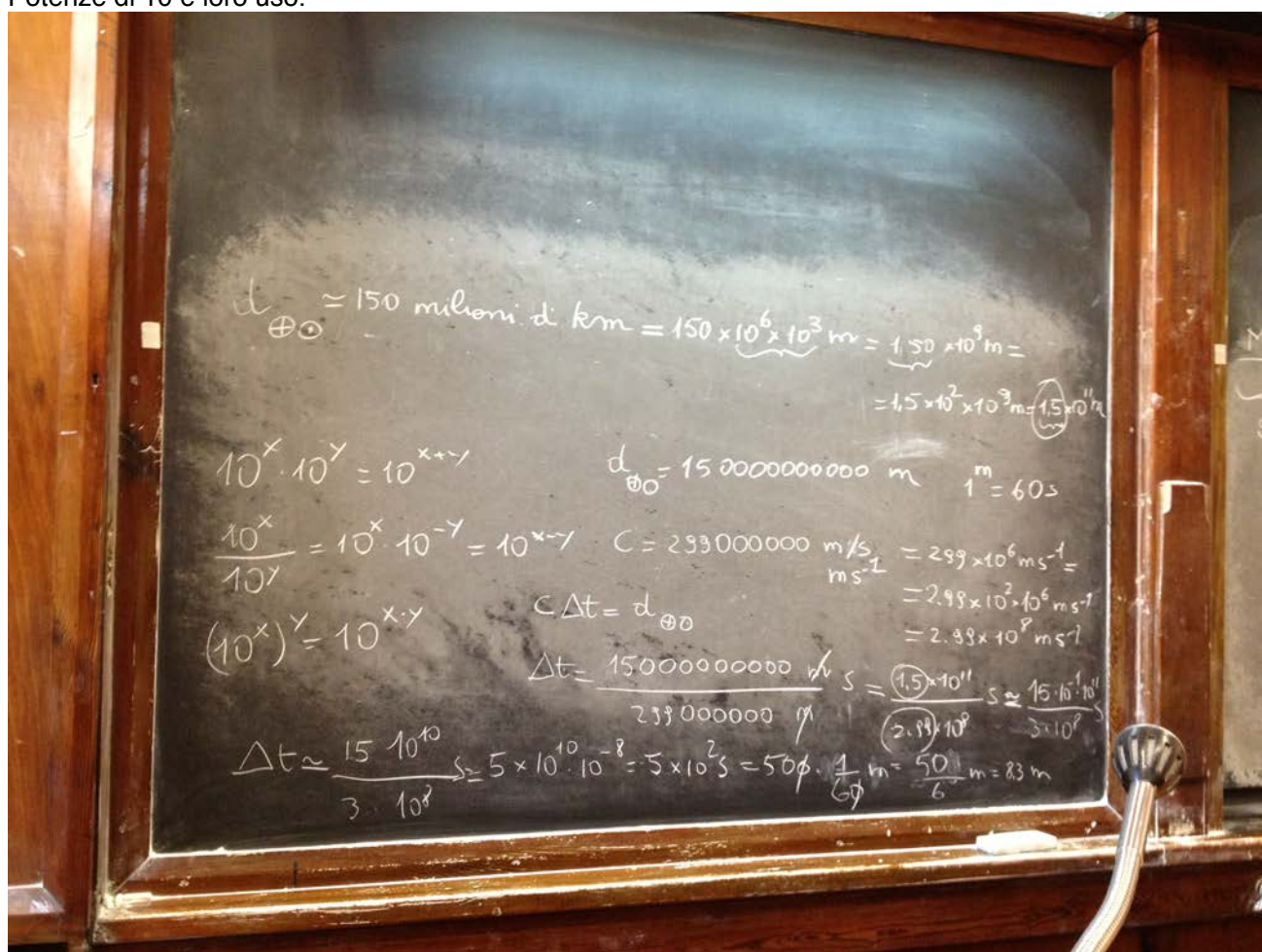


26 Settembre 2013

Grandezze fisiche, dimensioni e unità di misura.
Potenze di 10 e loro uso.



<p>MKS</p> <p><u> </u></p> <p>A</p> <p><u> </u></p> <p>SI</p>	10^{15} peta	1 000 000 000 000 000	
	10^{12} tera	1 000 000 000 000	
	10^9 giga	1 000 000 000	
	10^6 mega	1 000 000	
	10^3 kilo	1 000	
	10^2 atto	100	
	10^1 deca	10	K Kelvin
	$10^0 = 1$	1	Hz Hertz
	10^{-1} deci	0.1	kg
	10^{-2} centi	0.01	
	10^{-3} milli	0.001	
	10^{-6} (micro)	0.000001	

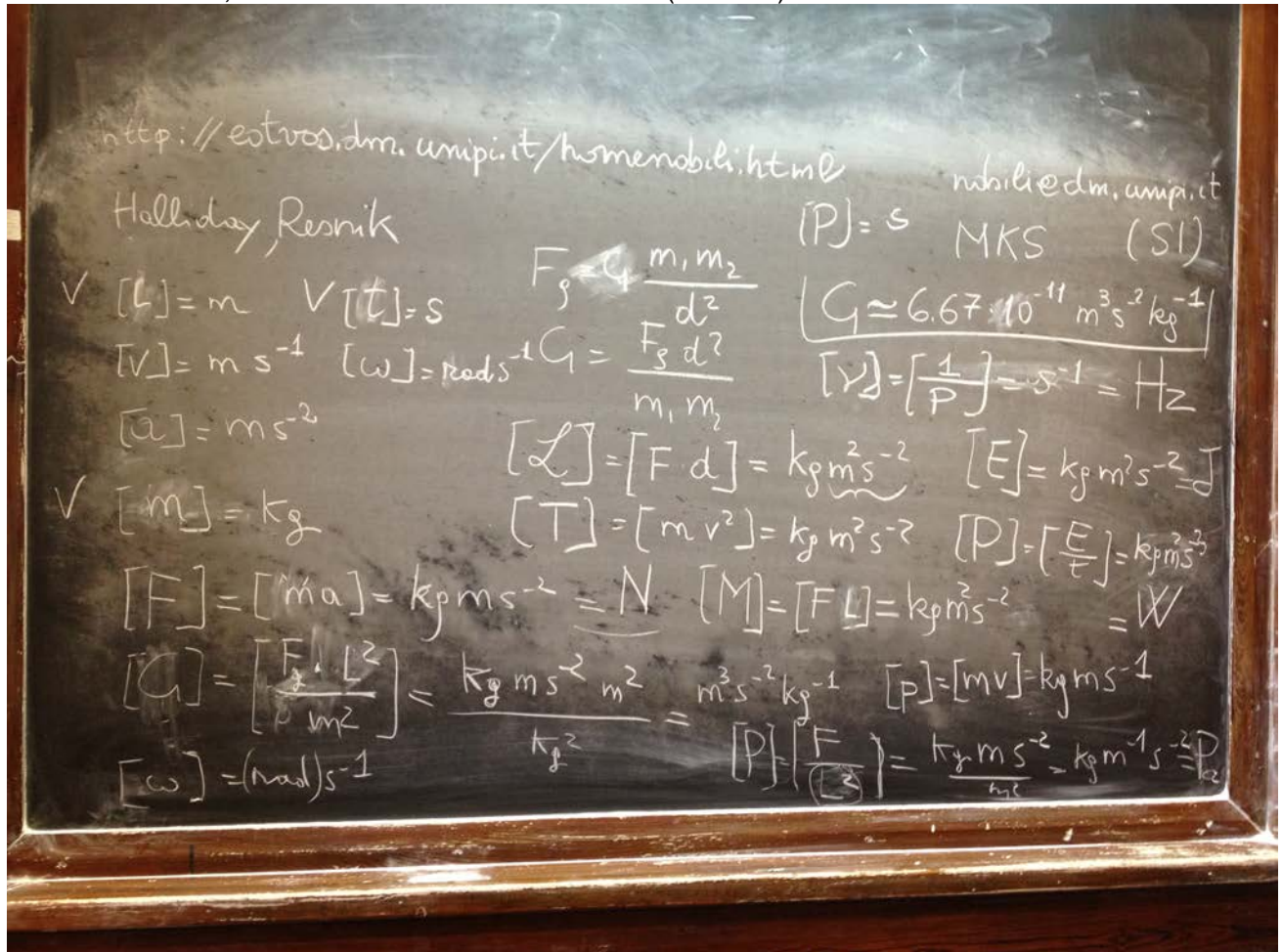
?

$$\begin{aligned}
 30 &= 330 \text{ m/s} = 3.30 \cdot 10^2 \frac{\text{m}}{\text{s}} = 3.30 \cdot 10^2 \frac{10^{-3} \text{ km}}{1/3600 \text{ hr}} \\
 &= 3.3 \cdot 10^4 \cdot 3600 \text{ km/hr} = \\
 &= 3.3 \cdot 10^4 \cdot 3.6 \cdot 10^3 \text{ km/hr} = 3.3 \cdot 3.6 \cdot 10^7 \text{ km/hr} \\
 &= 12 \cdot 10^7 \text{ km/hr} = 1.2 \cdot 10^8 \text{ km/hr} = \underline{1200 \text{ km/hr}}
 \end{aligned}$$

$$\begin{aligned}
 C &= 3 \cdot 10^8 \text{ m/s} = 3 \cdot 10^8 \cdot 10^{-3} \frac{\text{km}}{3600 \text{ hr}} = 3.36 \cdot 10^8 \text{ km/hr} \\
 &= 1.1 \cdot 10^9 \text{ km/hr}
 \end{aligned}$$

3 Ottobre 2013

Grandezze fisiche, dimensioni e verifiche dimensionali (continua)



$$\nu_{\oplus} = \underbrace{1.16 \times 10^{-5}}_{1/86400} \text{ Hz} \quad 1 \text{ yr} = 365.25 \text{ d} \quad d_{\oplus\text{Sun}} \approx 1.49 \times 10^{11} \text{ m}$$

$$= 365.25 \cdot 86400 \text{ s} = 3.65 \cdot 10^7 \cdot 8.64 \cdot 10^4 \text{ s} = 3.65 \cdot 8.64 \cdot 10^{11} \text{ s} \approx 3.15 \cdot 10^7 \text{ s}$$



$$v_{\oplus} \approx \frac{2\pi \cdot 1.49 \times 10^{11} \text{ m}}{3.15 \cdot 10^7 \text{ s}} \approx 3 \times 10^4 \text{ m s}^{-1} = 3 \cdot 10^4 \text{ m s}^{-1}$$

$$\approx 30 \text{ km s}^{-1}$$

$$P_{\oplus} \approx 3.15 \cdot 10^7 \text{ s} = 3 \cdot 10^7 \cdot 3.6 \times 10^3 \frac{\text{km}}{\text{h}} \approx 30 \cdot 3600 \frac{\text{km}}{\text{h}} \approx 3 \cdot 10^5 \frac{\text{km}}{\text{h}} \approx 10^5 \frac{\text{km}}{\text{h}}$$

$$\omega_{\oplus} = \frac{\alpha}{t} = \frac{2\pi}{P_{\oplus}} = 2\pi \nu_{\oplus}$$

$$P_{\oplus} = M_{\oplus} \nu_{\oplus}$$

[angolo] = adimensionale (radianti)



$$\nu = 0.2 \text{ Hz} \quad P = \frac{1}{\nu} = \frac{1}{0.2} \text{ s} = \frac{1}{2} \cdot 10 \text{ s} = 5 \text{ s}$$

$$\omega_{\text{rot}} = \frac{2\pi}{P} = 2\pi \nu \text{ rad s}^{-1} = 2\pi \cdot 0.2 \text{ rad s}^{-1} = 6.28 \cdot 2 \cdot 10^{-1} \text{ rad s}^{-1}$$

$$\approx 1.3 \times 10^{-1} \text{ rad s}^{-1} \approx 1.3 \text{ rad s}^{-1}$$

$$E_{\text{rot}} = \frac{1}{2} I \omega_{\text{rot}}^2$$

$$M_{\oplus} \approx 5.98 \cdot 10^{24} \text{ kg}$$



$$\frac{1}{2} I \omega^2$$

$$I = m l^2$$

10 Ottobre 2013

Grandezze fisiche, dimensioni e verifiche dimensionali (continua)

The chalkboard contains the following handwritten notes:

- $\vec{p} = m\vec{v}$
- $\vec{J} = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v}$
- Dimensions of angular momentum: $[\vec{J}] = \text{kg m ms}^{-1} = \text{kgm}^2\text{s}^{-1}$
- Dimensions of torque: $[\vec{N}] = \text{kgms}^{-2}$
- Diagram of a particle of mass m moving in a circular path with position vector \vec{r} and momentum vector \vec{p} .
- Diagram showing the derivative of position vector: $\frac{d\vec{r}}{dt} \equiv \vec{v}$
- Diagram illustrating the cross product of two vectors \vec{a} and \vec{b} , resulting in a vector \vec{c} perpendicular to the plane of \vec{a} and \vec{b} .
- Equation for the magnitude of the cross product: $|\vec{a} \times \vec{b}| = ab \sin \theta$
- Equation for the vector cross product: $\vec{c} = \vec{a} \times \vec{b}$
- Diagram illustrating the dot product of two vectors \vec{a} and \vec{b} , resulting in a scalar value $a \cdot b = b \cos \theta$.
- Calculation of Earth's orbital velocity: $v_{\oplus} = 30 \text{ km s}^{-1} = 30 \cdot 3600 \text{ km/h} \approx 3 \cdot 10^1 \cdot 3.6 \cdot 10^3 \text{ km/h} = 3.36 \cdot 10^4 \text{ km/h} \approx 10^5 \text{ km/h}$

<http://estros.dmm.unipi.it/horrendo.html>

$[T] = K$

$0^\circ C = +273 K$

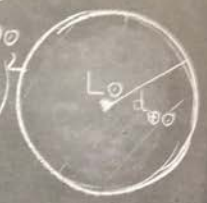
$g_\oplus = \frac{GM_\oplus}{R_\oplus^2}$
 $g_P = \frac{GM_P}{R_P^2}$
 $\frac{g_\oplus}{g_P} = \frac{M_\oplus R_P^2}{M_P R_\oplus^2} = 4\pi d_\oplus^2$
 $2\pi = \frac{\text{lunghezza circonferenza}}{\text{m raggio di cerchio}} = \frac{2\pi r}{r}$

$[\Phi] = \frac{\text{energia}}{s \cdot m^2} = \frac{\text{Potenza}}{\text{Area della superficie}} \approx 81 \cdot \left(\frac{1740}{6400}\right)^2$

$= \frac{J}{s \cdot m^2} = \frac{W}{m^2} = \frac{kg \cdot m^2 \cdot s^{-2}}{s \cdot m^2} = kg \cdot s^{-3}$
 $\phi = \frac{L_0}{4\pi d_\oplus^2}$

$\phi_0 \approx 13 \frac{KW}{m^2} = 13000 \frac{W}{m^2}$

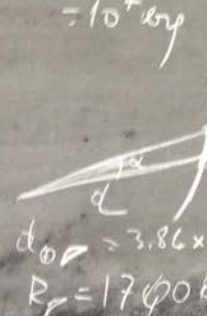
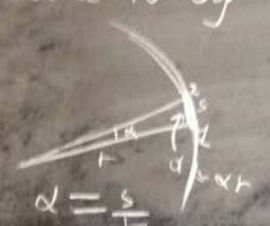
$[L_0] = W$



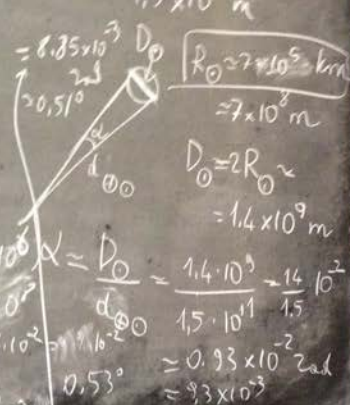
$L_\odot = 4\pi d_\oplus^2 \phi_0 \approx 4\pi \cdot (1.5 \cdot 10^8)^2 \cdot 13 \cdot 10^3 \frac{W}{m^2}$
 $\approx 1.2275 \cdot 10^{26} W = 3.5 \cdot 10^{26} \frac{J}{s}$

$1 \text{ Joule} = kg \cdot m^2 \cdot s^{-2} = 10^3 \cdot 10^4 \frac{g \cdot m^2}{s^2} \text{ MKS}$
 $= 10^7 \frac{g \cdot cm^2}{s^2} \text{ CGS}$
 $= 10^7 \text{ erg}$

$1 \text{ Joule} = 10^7 \text{ erg}$



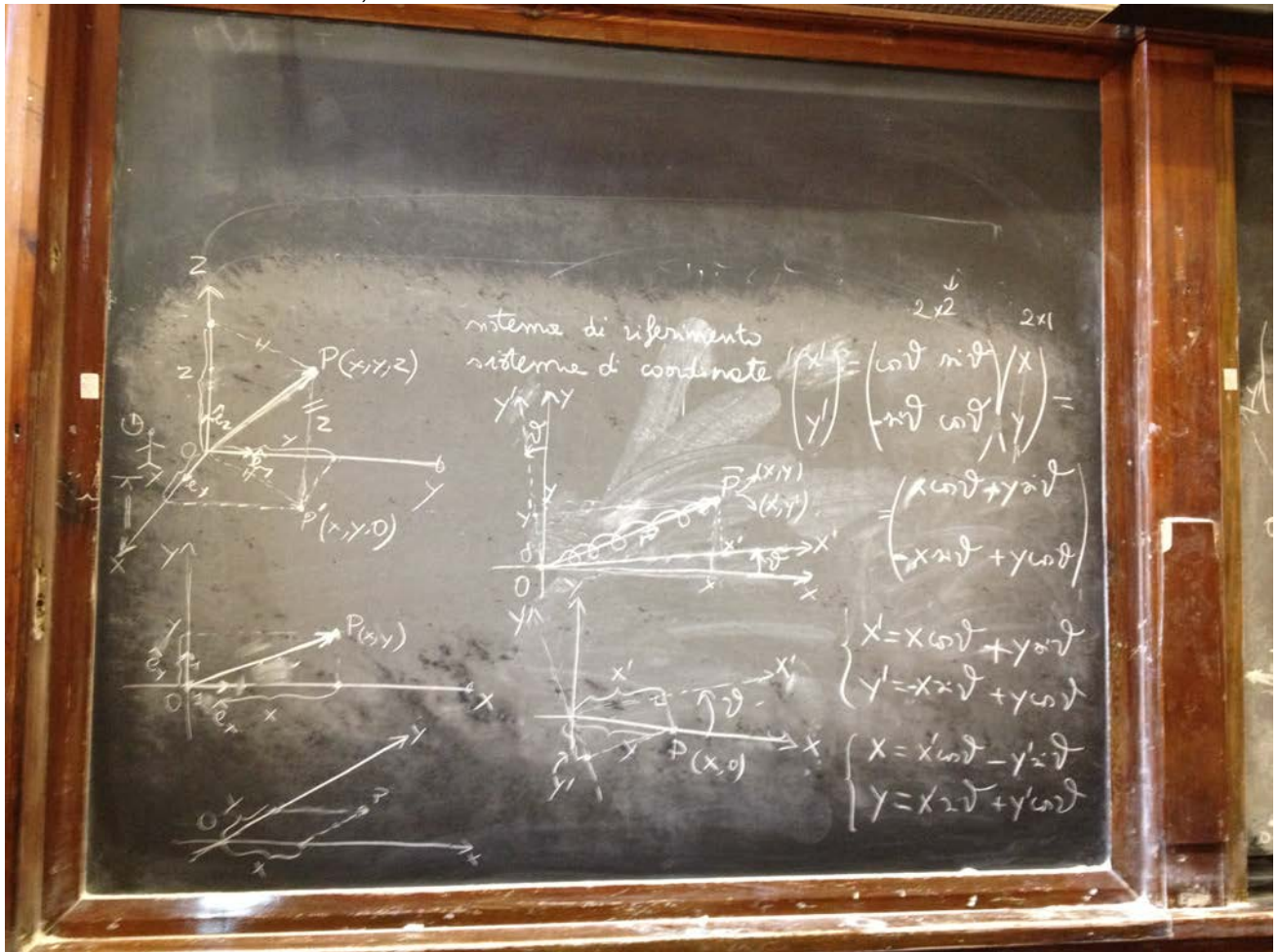
$d = \frac{2.17 \cdot 10^8}{3.86 \cdot 10^8} d_0$
 $d_0 = 3.86 \cdot 10^8 m = \frac{3.86 \cdot 10^8}{3.77} \cdot 10^2 = 1.02 \cdot 10^8 m$
 $R_P = 1740 km = 1.74 \cdot 10^6 m$



$R_0 \approx 6400 km$
 $1.5 \cdot 10^{11} m$
 $R_0 \approx 7 \cdot 10^5 km = 7 \cdot 10^8 m$
 $D_0 = 2R_0 \approx 1.4 \cdot 10^9 m$
 $\alpha = \frac{D_0}{d_0} = \frac{1.4 \cdot 10^9}{1.5 \cdot 10^{11}} = \frac{14}{15} \cdot 10^{-2} \approx 0.93 \cdot 10^{-2} rad = 9.3 \cdot 10^{-3}$

24 Ottobre 2013

Sistemi di coordinate e vettori; modulo e somma



$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} X' \\ Y' \end{pmatrix}$$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$
 $\sin^2\theta + \cos^2\theta = 1$

$Oxyz \rightarrow O'x'y'z'$
 $\Delta r = r_2 - r_1$
 $\Delta s = \Delta t$
 $v = \frac{\Delta s}{\Delta t}$
 $[k] = \frac{3 \times 1}{3 \times 1}$

$X = X' \cos\theta - Y' \sin\theta$
 $Y = X' \sin\theta + Y' \cos\theta$
 $z = z'$

$\vec{a} \cdot \vec{a} = a_x^2 + a_y^2 + a_z^2$

$\vec{a} = (a_x, a_y)$
 $\vec{b} = (b_x, b_y)$
 $\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y)$
 $\vec{a} - \vec{b} = \vec{a} + (-1\vec{b})$

$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$
 $a_1 b_1 + a_2 b_2$

$\vec{r} = x\hat{e}_x + y\hat{e}_y$
 $\vec{r} = (x, y)$

$\vec{a} + \vec{b} = \vec{b} + \vec{a}$
 $d\vec{a} = \vec{a} a$
 $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$

$\vec{r} = (x, y)$
 $\vec{P} = (x, y)$

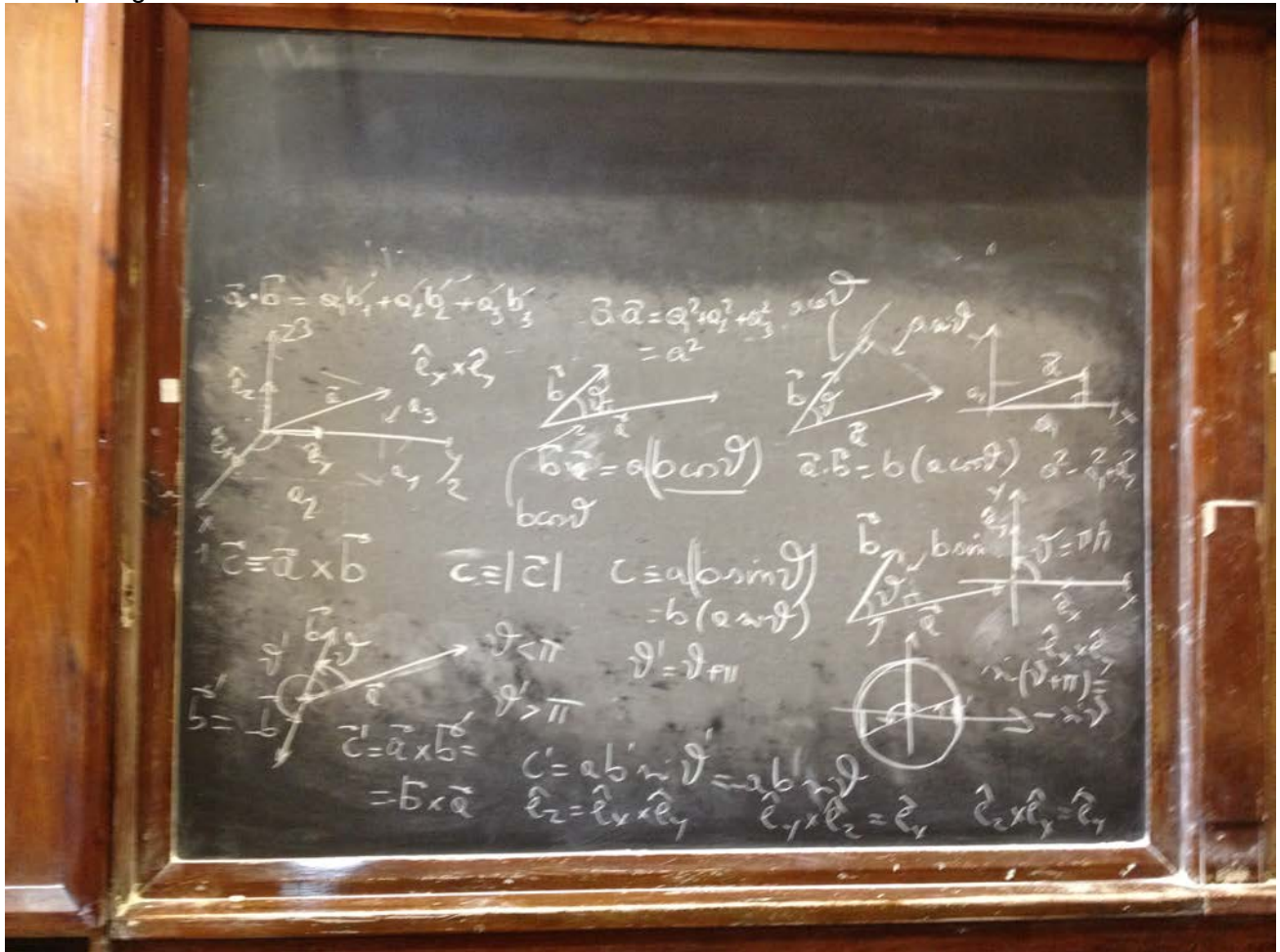
Presence 82

31 Ottobre 2013

Prodotto scalare e prodotto vettore. Definizione e rappresentazione grafica

Perché i vettori?

Esempi di grandezze fisiche vettoriali



$\vec{a} = (a_1, a_2, a_3)$ $\vec{b} = (b_1, b_2, b_3)$ $\vec{c} = \vec{a} \times \vec{b}$ $\vec{c} = (c_1, c_2, c_3)$
 $\hat{e}_z = \hat{e}_x \times \hat{e}_y$ $\alpha \cdot mv^2 = m \vec{v} \cdot \vec{v} = m(v_1^2 + v_2^2 + v_3^2)$

$c_1 \equiv a_2 b_3 - a_3 b_2$
 $c_2 \equiv a_3 b_1 - a_1 b_3$
 $c_3 \equiv a_1 b_2 - a_2 b_1$



$\Delta L = \vec{F} \cdot \Delta \vec{s}$
 $F \Delta s \cos \theta = F \Delta s \sin \theta$

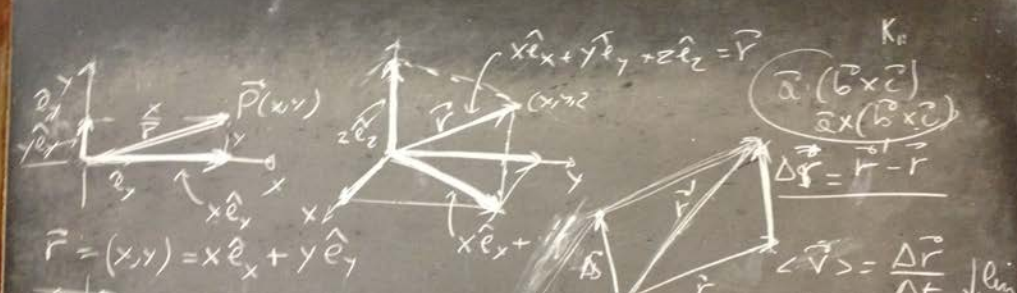
$\vec{M} = \vec{r} \times \vec{F}$ $\vec{M} = (0, 0, M)$



$J = r p = r m v$ $\vec{p} = m \vec{v}$

$\vec{b} = (0, b, 0)$
 $\vec{F} = (F, 0, 0)$
 $\vec{M} = (0, 0, M)$

$\vec{J} \equiv \vec{r} \times \vec{p}$



$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{a} \times (\vec{b} \times \vec{c})$
 $\Delta \vec{r} = \vec{r}' - \vec{r}$

$\vec{r} = (x, y) = x \hat{e}_x + y \hat{e}_y$

$\langle \vec{v} \rangle = \frac{\Delta \vec{r}}{\Delta t}$ $\vec{v} = \frac{d\vec{r}}{dt}$



$\frac{\Delta \vec{s}}{\Delta t} = \frac{\vec{r}' - \vec{r}}{\Delta t} = \frac{\Delta \vec{r}}{\Delta t}$
 $\langle \vec{v} \rangle = \frac{\Delta \vec{s}}{\Delta t}$

21
24
37
82

$\langle v \rangle = \frac{\Delta s}{\Delta t}$

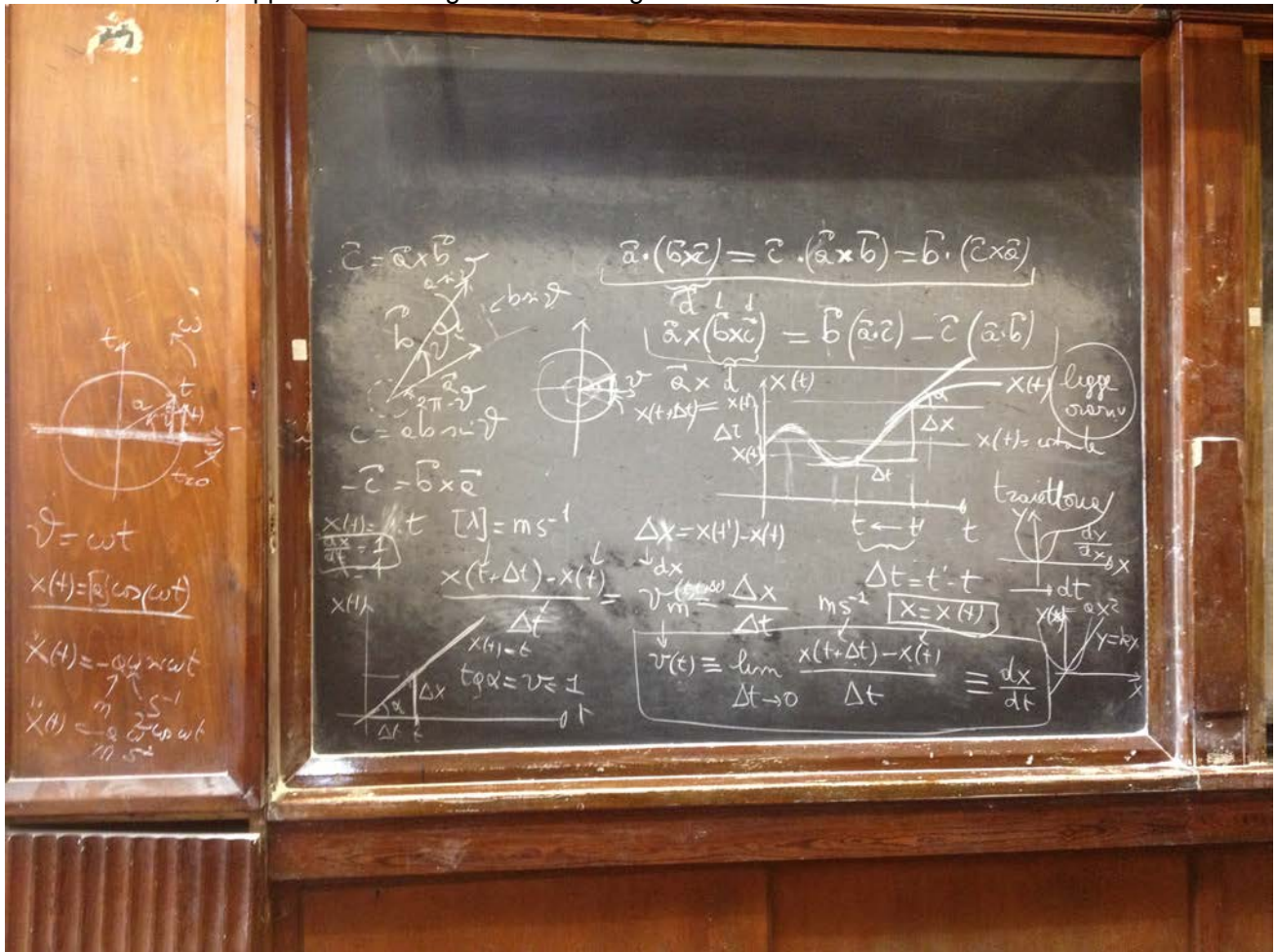


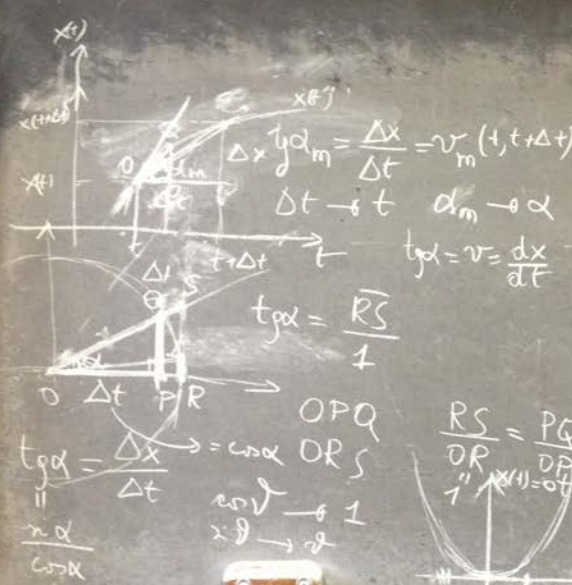
$\vec{a} = \frac{d\vec{v}}{dt}$

7 Novembre 2013

Ancora sui vettori

Derivate: richiami, rappresentazione geometrica e significato fisico





$$x(t+\Delta t) = \lambda \cdot (t+\Delta t)$$

$$x(t) = \lambda t \quad \lambda = \text{ms}^{-1}$$

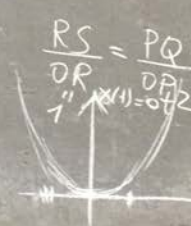
$$v(t) = \lim_{\Delta t \rightarrow 0} \frac{\lambda(t+\Delta t) - \lambda t}{\Delta t} = \lambda$$

$$= \lambda \lim_{\Delta t \rightarrow 0} \frac{t + \lambda \Delta t - t}{\Delta t} = \lambda$$

$$= \lambda \lim_{\Delta t \rightarrow 0} 1 = \lambda$$

$$x = \lambda t \quad \frac{dx}{dt} = \lambda$$

$$\frac{d}{dx}(\lambda x) = \lambda$$



$$x(t) = at^2 \quad [a] = \text{ms}^{-2}$$

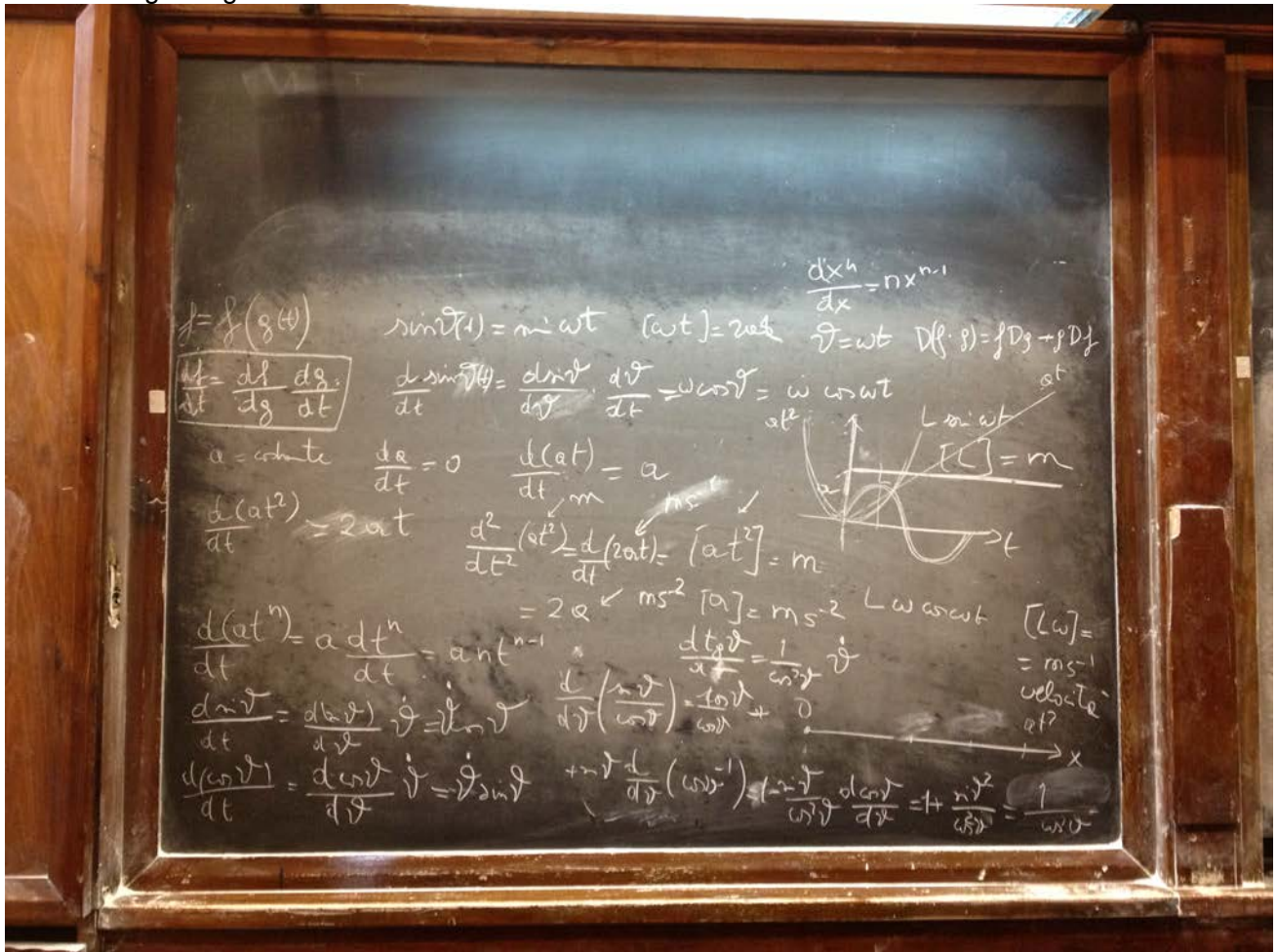
$$\frac{dx}{dt} = \lim_{\Delta t \rightarrow 0} \frac{a(t+\Delta t)^2 - at^2}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{a(t^2 + 2t\Delta t + \Delta t^2) - at^2}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{2at\Delta t + a\Delta t^2}{\Delta t} = \lim_{\Delta t \rightarrow 0} (2at + a\Delta t) = 2at$$

$$\frac{dx}{dt} = 2at$$

$f(t), g(t) \quad D \equiv \frac{d}{dt}$
 $D(f(t) + g(t)) = Df(t) + Dg(t)$
 $D(af(t)) = a Df(t) \quad D(af(t) + bg(t)) = a Df(t) + b Dg(t)$
 $D(f(t)g(t)) = f(t)Dg(t) + g(t)Df(t)$
 $D\left(\frac{f(t)}{g(t)}\right) = \frac{g(t)Df(t) - f(t)Dg(t)}{g(t)^2}$
 $D(f(t)^2) = \frac{d}{dt} f(t)^2 = 2f(t) \frac{df}{dt} = 2af(t)$

$x(t) = a \cos \omega t$
 $v(t) = \lim_{\Delta t \rightarrow 0} \frac{a \cos \omega(t+\Delta t) - a \cos \omega t}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{a(\cos \omega t \cos \omega \Delta t - \sin \omega t \sin \omega \Delta t) - a \cos \omega t}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{a \cos \omega t (\cos \omega \Delta t - 1) - a \sin \omega t \sin \omega \Delta t}{\Delta t}$

21 Novembre 2013
 Ancora sulle derivate
 Richiami sugli integrali



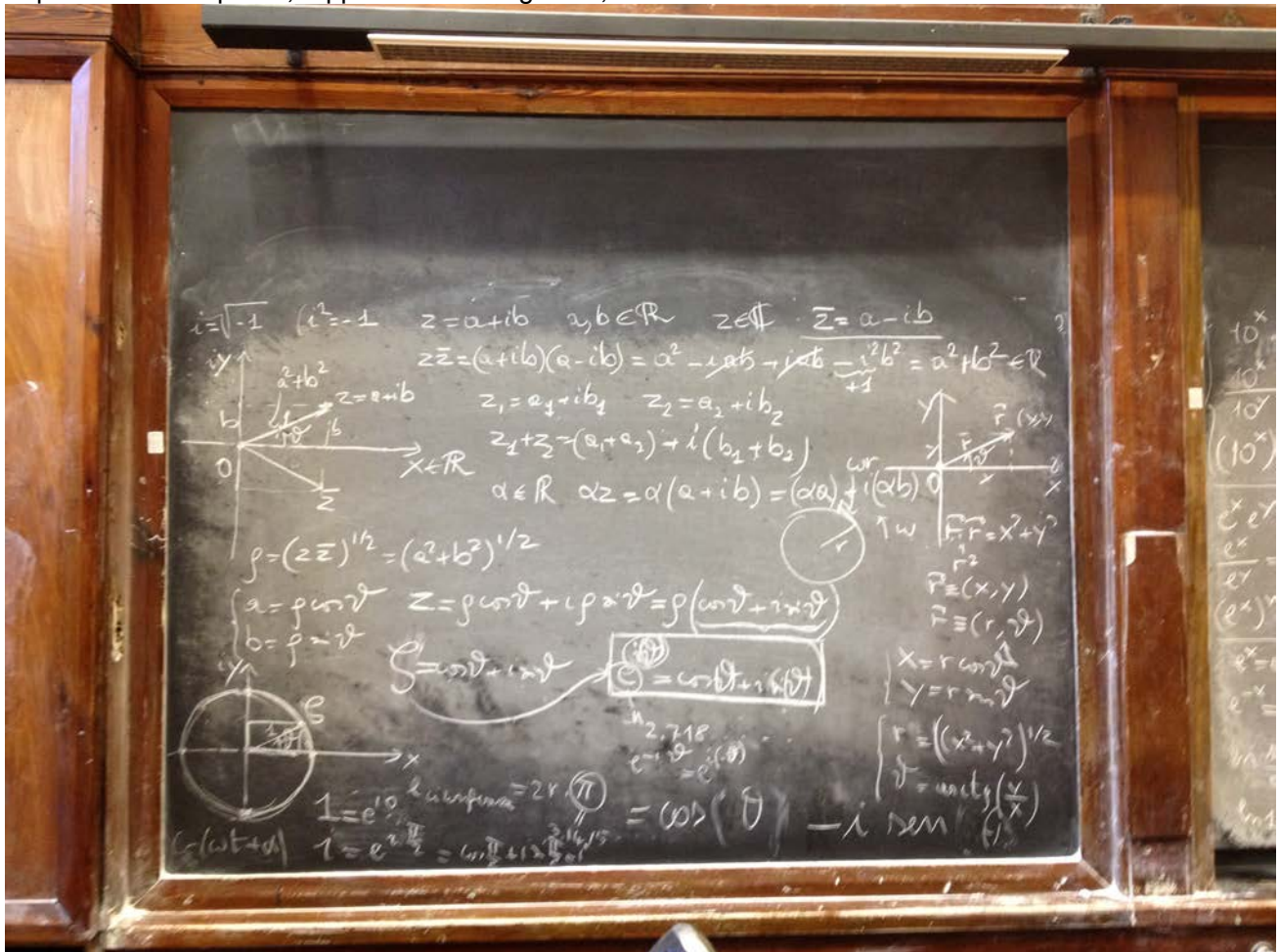
$v = v(t)$
 $x = x(t)$
 $\frac{dx(t)}{dt} = v(t)$
 $dx = v(t) dt$
 $\frac{d(x(t)+c)}{dt} = v(t)$
 $\frac{d}{dt} = \frac{d}{dt} + \frac{dc}{dt}$
 $\Delta x = \int dx = \int_{t_0}^{t_1} v(t) dt$
 \sum_{t_0, t_1}
 $F(t) = \int f(t) dt$
 $F(t) = \int f(t) dt + C(t)$
 $\int (f(t) \cdot g(t)) dt = F(t) \cdot g(t) - \int F(t) \frac{dg}{dt} dt$
 $\int a f(t) dt = a \int f(t) dt = a F(t)$

$\int f(t)g(t)dt = f(t)C(t) - \int C(t)\frac{df}{dt}dt = g(t)F(t) - \int F(t)\frac{dg}{dt}dt$
 $t = \phi(u) \quad dt = \frac{d\phi(u)}{du} du$
 $\int f(t)dt = \int (f(\phi(u)) \frac{d\phi(u)}{du}) du$
 $\int at = \frac{at^2}{2} + c$
 $\int t^n dt = \frac{t^{n+1}}{n+1} + c$
 $\int \frac{dt}{t} = \ln|t| + c$
 $\frac{1}{\omega} \frac{d(\omega t)}{dt} = 1$
 $\int \frac{1}{\omega} d(\omega t) = \frac{1}{\omega} \omega t = t$
 $\int \frac{1}{\omega} d(\omega t) = \frac{1}{\omega} \ln|\omega t|$
 $\int \cos \omega t dt = \frac{1}{\omega} \sin \omega t$
 $\int \sin \omega t dt = -\frac{1}{\omega} \cos \omega t$
 $\int \frac{1}{\omega} d(\omega t) = \frac{1}{\omega} \ln|\omega t|$
 $e^{i\omega t} = \cos \omega t + i \sin \omega t$
 $v^2 = -1$
 $\int \frac{dt}{t} = \ln|t| + c$
 $\int \frac{1}{\omega} d(\omega t) = \frac{1}{\omega} \ln|\omega t|$
 $\int \frac{1}{\omega} d(\omega t) = \frac{1}{\omega} \ln|\omega t|$
 $\int \frac{1}{\omega} d(\omega t) = \frac{1}{\omega} \ln|\omega t|$

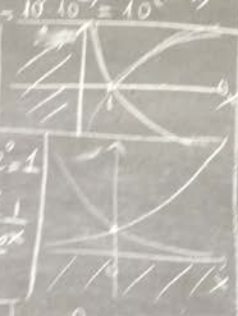
28 Novembre 2013

Richiami sui numeri complessi. Perché i numeri complessi?

Esponenziali complessi, rappresentazione grafica, relazione con i vettori e utilizzo in fisica.



$10^x \cdot 10^y = 10^{(x+y)}$ $10^0 = 1$
 $\frac{10^x}{10^y} = 10^x \cdot \frac{1}{10^y} = 10^x \cdot 10^{-y} = 10^{(x-y)}$
 $(10^x)^y = 10^{xy}$
 $e^x e^y = e^{(x+y)}$ $e^0 = 1$
 $\frac{e^x}{e^y} = e^{(x-y)}$ $e^{-x} = \frac{1}{e^x}$
 $(e^x)^y = e^{xy}$
 $e^x = a$ $e^y = b$ $x = \ln a$ $y = \ln b$
 $e^{-x} = \frac{1}{e^x} = \frac{1}{a}$ $\ln \frac{1}{a} = -x$
 $\ln 1 = 0$



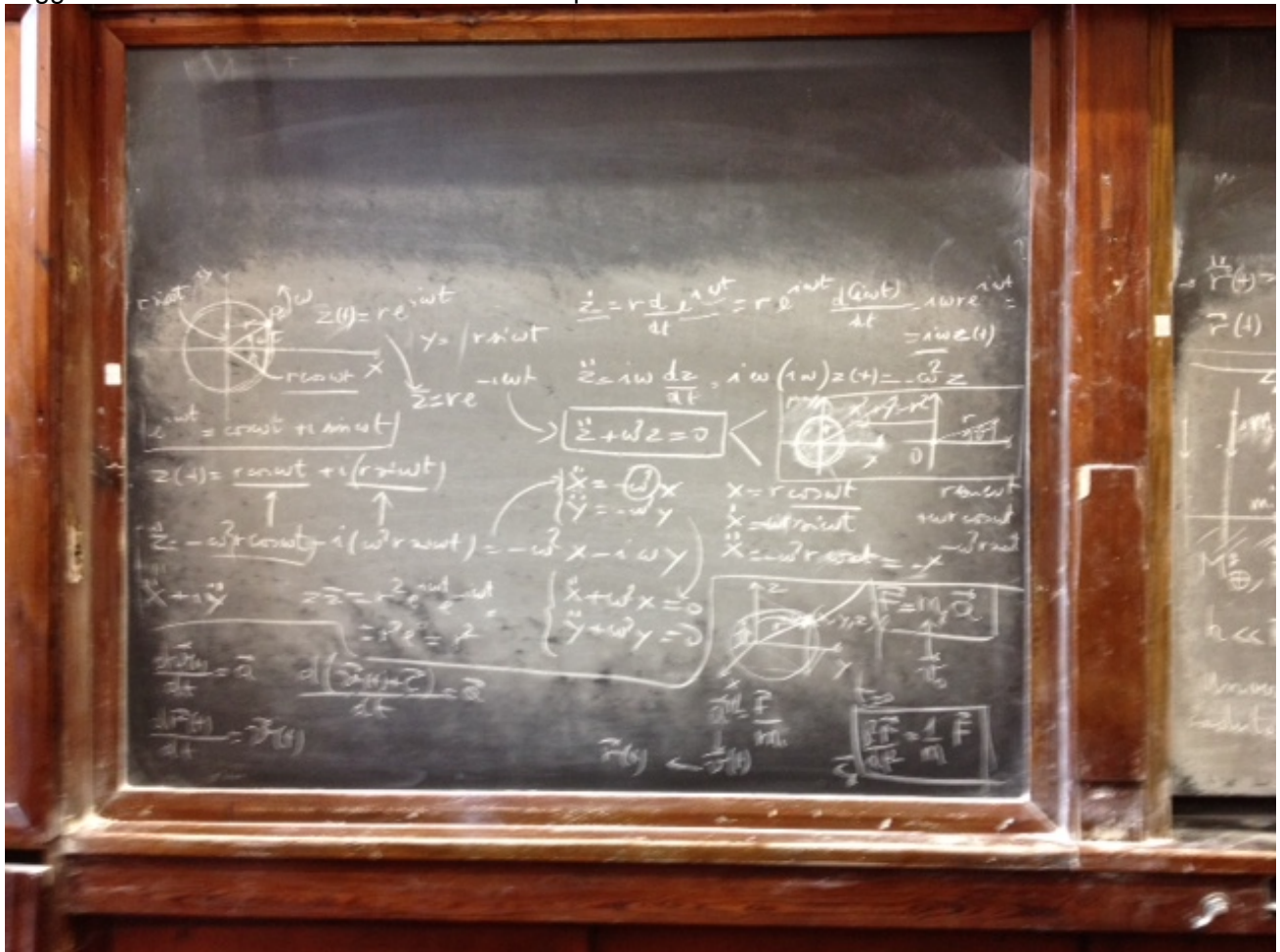
$10^x = a$ $x = \log_{10} a$
 $10^y = b$ $y = \log_{10} b$
 $\frac{10^x \cdot 10^y}{ab} = \frac{10^{(x+y)}}{ab}$ $x+y = \log_{10}(ab)$
 $\log_{10} a = \log_{10} b$
 $10^{\frac{1}{x}} = \frac{1}{10^{\frac{1}{x}}} = \frac{1}{b}$ $\log_{10} \frac{1}{b} = -x$
 $10^x \cdot 10^{-y} = 10^{(x-y)}$ $\log_{10} \frac{a}{b} = x-y = \log_{10} a - \log_{10} b$
 $\log_{10} \frac{1}{b} = -x$ $\log_{10} 1 = 0$
 $\log_{10} 1 = \log_{10} b$

$\frac{d e^x}{dx} = e^x$ $\frac{d e^{ax}}{dx} = \frac{d e^{ax}}{d(ax)} \cdot \frac{d(ax)}{dx} = a e^{ax}$ $\frac{d(\ln x)}{dx} = \frac{1}{x}$ K_e
 $e^{i\theta} = \cos(\theta) + i \sin(\theta)$
 $e^{-i\theta} = \cos(-\theta) + i \sin(-\theta) = \cos(\theta) - i \sin(\theta)$
 $e^{-i\theta} = \frac{1}{e^{i\theta}} = \frac{1}{\cos(\theta) + i \sin(\theta)} \cdot \frac{\cos(\theta) - i \sin(\theta)}{\cos(\theta) - i \sin(\theta)} = \frac{\cos(\theta) - i \sin(\theta)}{\cos^2(\theta) + \sin^2(\theta)} = \frac{\cos(\theta) - i \sin(\theta)}{1}$
 $e^{i\theta} = \cos(\theta) + i \sin(\theta)$ $\theta = \omega t$ $[\omega] = \text{rad s}^{-1}$
 $e^{i\theta} = \cos(\theta) + i \sin(\theta)$
 $\frac{d e^{i\theta}}{d\theta} = \frac{d(\cos(\theta) + i \sin(\theta))}{d\theta} = -\sin(\theta) + i \cos(\theta) = i(\cos(\theta) + i \sin(\theta)) = i e^{i\theta}$
 $\frac{d e^{i\omega t}}{dt} = i \omega e^{i\omega t}$

5 Dicembre 2013

Ancora sui numeri complessi

Legge fondamentale di Newton e concetto di equazione del moto



$\dot{r}(t) = \frac{1}{m} \vec{F}(t)$ eq. del moto
 $\vec{r}(t) = ?$ legge newtoniana

$\vec{r}(t) = \vec{r}(0) + \vec{v}(0)t + \frac{1}{2} \vec{a} t^2$ condizioni iniziali

$\vec{F}_g = G \frac{M_1 m_2}{r^2}$

$\vec{F}_{C\oplus} = -G \frac{M_{\oplus} m_2}{R_{\oplus}^2}$
 \parallel
 $m_2 a = -G \frac{M_{\oplus} m_2}{R_{\oplus}^2}$
 $a = -\frac{G M_{\oplus}}{R_{\oplus}^2} = -9.8 \text{ ms}^{-2}$

e norme (leggi) gravitazionale
 $h \ll R_{\oplus}$
 Universalità della caduta libera
 principio di equivalenza tra massa gravitazionale

$m \ddot{z} = -G \frac{M m}{r^2}$
 $\ddot{z} = -\frac{g}{r^2}$
 $\dot{z}(t=t_0) = v_0$
 $z(t) = \int \dot{z} dt$
 $z(t=t_0) = h$
 $\dot{z}(t_f) = 0$

$\ddot{z} = -\left(\frac{GM_{\oplus}}{R_{\oplus}^2}\right) \frac{R_{\oplus}^2}{z^2}$

$\ddot{z} = -g$

$\int \ddot{z} dt = \dot{z} = -gt + v_0$

$\dot{z}(t=t_0) = v_0 = 0$

$\int \dot{z} dt = z = -\frac{1}{2}gt^2 + v_0 t + z_0$

$z(t=t_0) = z_0 = h$

$z(t_f) = 0 = h - \frac{1}{2}gt_f^2$

$h = \frac{1}{2}gt_f^2 \Rightarrow t_f = \sqrt{\frac{2h}{g}}$

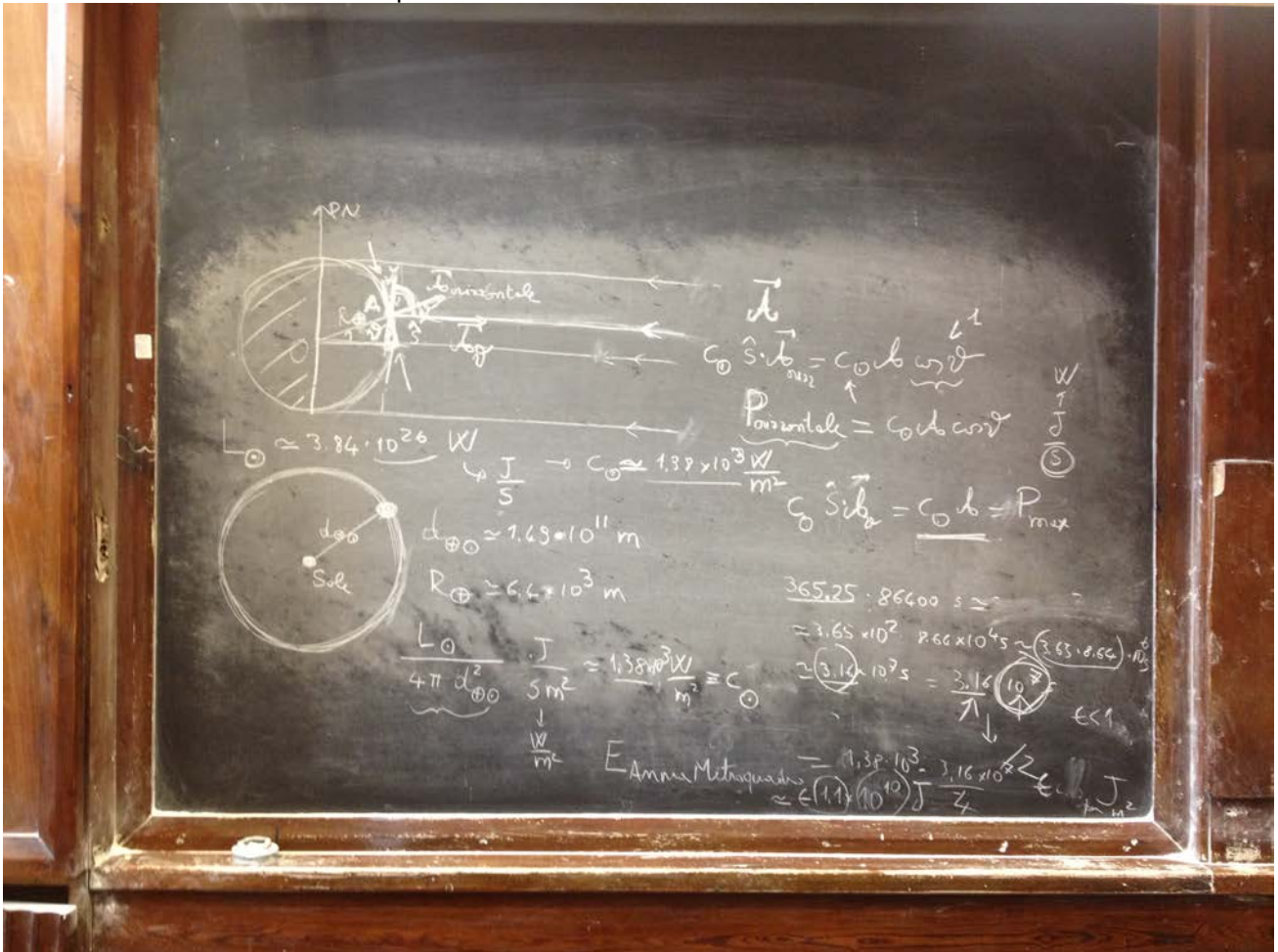
$\dot{z}(t_f) = 0 = -gt_f + v_0$

$v_0 = gt_f = g \sqrt{\frac{2h}{g}} = \sqrt{2gh}$

12 Dicembre 2013: 1o compito

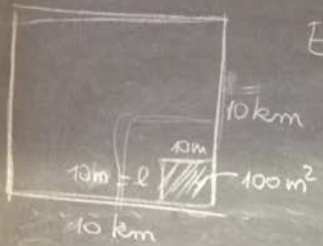
19 Dicembre 2013

Soluzione e discussione del compito del 12 Dicembre



$$4\pi R_{\oplus}^2 = 4\pi (6.4)^2 \cdot 10^{12} \text{ m}^2 = 5.14 \cdot 10^{14} \text{ m}^2$$

$$R_{\oplus} = 6400 \text{ km} = 6.4 \cdot 10^6 \text{ m}$$



$$10^4 \text{ m} \cdot 10^4 \text{ m} = 10^8 \text{ m}^2$$

$$E_{\text{Amma Terra}} \approx E_{\text{Amma Metroquadro}} \cdot \frac{5.14 \cdot 10^{14} \cdot 10^{-6}}{54 \cdot 10^8 \text{ m}^2}$$

$$\approx 1.1 \times 10^{10} \cdot 5.14 \cdot 10^8 \text{ J} \approx 5.74 \cdot 10^{18} \text{ J}$$

$$\frac{10^2 \text{ m}^2}{10^8 \text{ m}^2} = 10^{-6}$$

$$\approx 15\% = \frac{15}{100} = 0.15 = 1.5 \cdot 10^{-1} \approx 8.5 \cdot 10^{17} \text{ J}$$

K_{II}

70% acqua
30% terraemera

$$P_{\text{Consomma Terra}} \approx 1.5 \cdot 10^{13} \frac{\text{J}}{\text{s}}$$

$$E_{\text{Consomma Terra}} \approx 1.5 \cdot 10^{13} \cdot 3.16 \cdot 10^2 \text{ J} \approx 4.7 \cdot 10^{20} \text{ J}$$

$$100 \text{ m}^2 \cdot \frac{100}{30} = \frac{10^4}{30} = \frac{10^3}{3} \approx 333 \text{ m}^2$$

$$l \rightarrow l' = \sqrt{333} \approx 18 \text{ m}$$

$$50\% \quad 100 \text{ m}^2 \cdot \frac{100}{50} = \frac{10^4}{5} = \frac{10^3}{5} = 200 \text{ m}^2$$

$$l' = \sqrt{200} = \sqrt{2} \cdot 10 \text{ m}$$

$$\frac{E_{\text{Amma Terra}}}{E_{\text{Consomma Terra}}} \approx \frac{8.5 \cdot 10^{17}}{4.7 \cdot 10^{20}} \approx 1.8 \cdot 10^{-3}$$

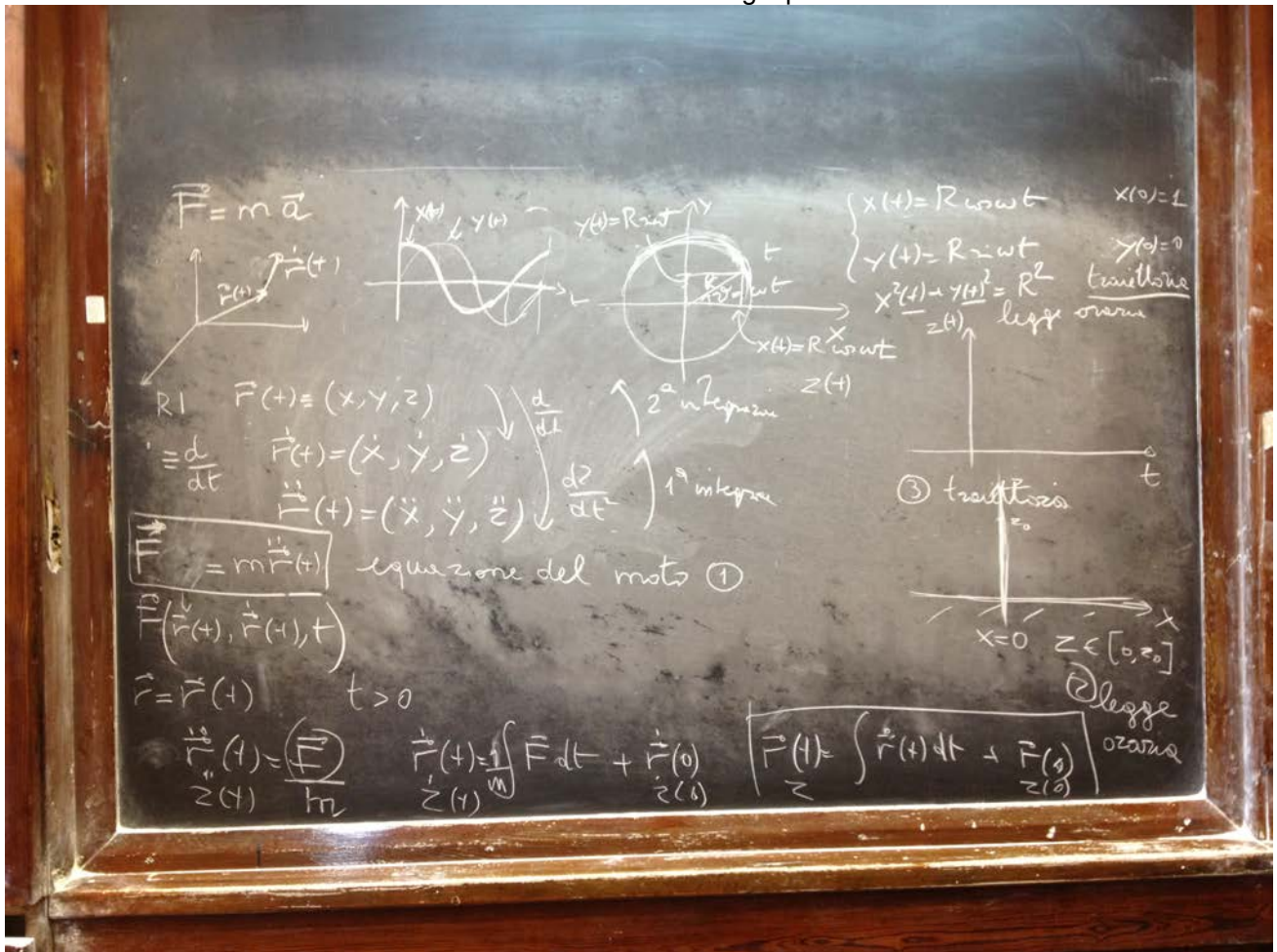
$$\approx 2\%$$


11 Febbraio 2014

Equazioni del moto di corpi puntiformi in un riferimento inerziale.

Integrazione delle equazioni del moto. Concetti di legge oraria e di traiettoria.

Definizione di lavoro. Forze conservative e definizione di energia potenziale.





$$U(P) - U(P_0) = - \int_{P_0}^P \vec{F}_c \cdot d\vec{r} = - m \int_{P_0}^P \ddot{\vec{r}} \cdot d\vec{r} = - m \int_{P_0}^P \dot{\vec{v}} \cdot d\vec{r}$$

$$= - m \int_{t_0}^t \dot{\vec{v}} \cdot \vec{v} dt = - m \int_{t_0}^t \frac{d}{dt} \left(\frac{1}{2} m v^2 \right) dt = - \left(\frac{1}{2} m v^2 \right)_0^t$$

$$\frac{d}{dt} v^2 = \frac{d}{dt} (\vec{v} \cdot \vec{v}) = \dot{\vec{v}} \cdot \vec{v} + \vec{v} \cdot \dot{\vec{v}} = 2 \dot{\vec{v}} \cdot \vec{v}$$

\vec{F}_c forza conservativa

$$U(P) - U(P_0) = - \int_{P_0}^P \vec{F}_c \cdot d\vec{r} \quad U \text{ energia potenziale}$$

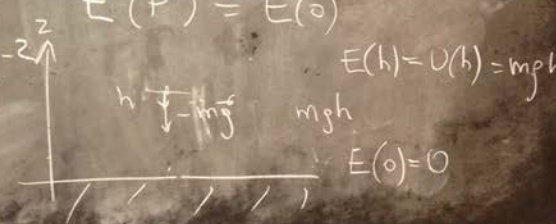
$$U(P) - U(P_0) = - \int_{P_0}^P \vec{F}_c \cdot d\vec{r} = - \left(m \int_{P_0}^P \ddot{\vec{r}} \cdot d\vec{r} = - m \int_0^t \dot{\vec{v}} \cdot \vec{v} dt = - \int_0^t \frac{d}{dt} \left(\frac{1}{2} m v^2 \right) dt \right)$$

$$\dot{\vec{r}} = \vec{v} \quad T = \frac{1}{2} m v^2 \quad = - \int_0^t d \left(\frac{1}{2} m v^2 \right) = - \left(\frac{1}{2} m v^2 \right)_0^t = T_0 - T(P)$$

$$\frac{d\vec{r}}{dt} = \vec{v} \quad \Rightarrow U(P) + T(P) = U(P_0) + T(0)$$

$$\frac{d}{dt} (v^2) = \frac{d}{dt} (\vec{v} \cdot \vec{v}) = 2 \dot{\vec{v}} \cdot \vec{v} \quad \parallel \quad E(P) = E(0)$$

$[J] = \text{kg m s}^{-2} \cdot \text{m} = \text{kg m}^2 \text{s}^{-2}$

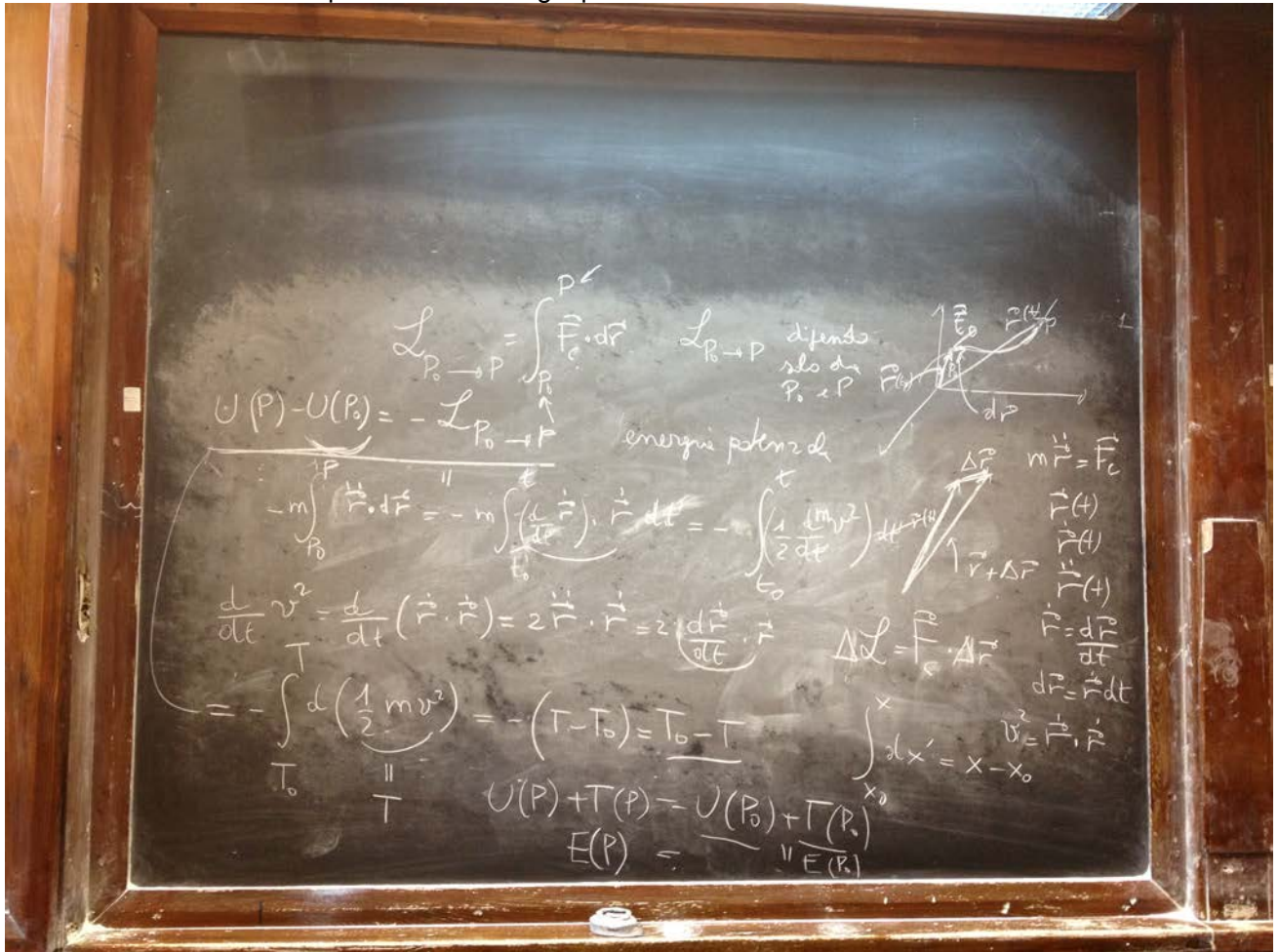


$E(h) = U(h) = mgh$
 $E(0) = 0$

13 Febbraio 2014

Energia potenziale. Scelta del livello di riferimento.

Calcolo della energia potenziale gravitazionale di un corpo sulla superficie della Terra oppure molto lontano da essa: confronto del segno e del valore e dimostrazione che non c'è contraddizione tra le formule usate nei due casi per definire l'energia potenziale.



$U(z) - U(z_0) = - \int_{z_0}^z \vec{F} \cdot d\vec{s} = - \int_{z_0}^z -mg \, dz = +mgz$
 $U(z) = U(z_0) + mgz$
 $U(z) - U(z_0) = - \int_{z_0}^z \vec{F} \cdot d\vec{s} = - \int_{z_0}^z -mg \, dz = +mgz$
 $U(z) = U(z_0) + mgz$

$U(r) - U(r_0) = - \int_{r_0}^r \vec{F} \cdot d\vec{r} = - \int_{r_0}^r -\frac{GMm}{r^2} \, dr = -GMm \left[\frac{1}{r} \right]_{r_0}^r = -GMm \left[\frac{1}{r} - \frac{1}{r_0} \right]$
 $U(r) = U(r_0) - GMm \left[\frac{1}{r} - \frac{1}{r_0} \right]$

$U(R_0+z) = -\frac{GMm}{R_0+z} = -\frac{GMm}{R_0(1+\frac{z}{R_0})} = -\frac{GMm}{R_0} \left(\frac{1}{1+\epsilon} \right) = -\frac{GMm}{R_0} \left(\frac{m}{R_0} \right) \frac{m}{z}, R_0 \quad z \ll R_0$
 $\epsilon = \frac{z}{R_0} \ll 1$
 $\frac{1}{1+\epsilon} = 1 - \epsilon + \frac{1}{2}\epsilon^2 - \frac{1}{6}\epsilon^3 + \dots$
 $U(R_0+z) = -\frac{GMm}{R_0} \left(1 - \epsilon + \frac{1}{2}\epsilon^2 - \frac{1}{6}\epsilon^3 + \dots \right) = -\frac{GMm}{R_0} + \frac{GMm}{R_0} \epsilon - \frac{1}{2} \frac{GMm}{R_0} \epsilon^2 + \frac{1}{6} \frac{GMm}{R_0} \epsilon^3 - \dots$
 $U(R_0+z) = -\frac{GMm}{R_0} + \frac{GMm}{R_0} \frac{z}{R_0} - \frac{1}{2} \frac{GMm}{R_0} \left(\frac{z}{R_0} \right)^2 + \frac{1}{6} \frac{GMm}{R_0} \left(\frac{z}{R_0} \right)^3 - \dots$
 $U(R_0+z) = -\frac{GMm}{R_0} + \frac{GMm}{R_0} \frac{z}{R_0} - \frac{1}{2} \frac{GMm}{R_0} \frac{z^2}{R_0^2} + \frac{1}{6} \frac{GMm}{R_0} \frac{z^3}{R_0^3} - \dots$
 $U(R_0+z) = -\frac{GMm}{R_0} + \frac{GMm}{R_0} \frac{z}{R_0} - \frac{1}{2} \frac{GMm}{R_0} \frac{z^2}{R_0^2} + \frac{1}{6} \frac{GMm}{R_0} \frac{z^3}{R_0^3} - \dots$
 $U(R_0+z) = -\frac{GMm}{R_0} + \frac{GMm}{R_0} \frac{z}{R_0} - \frac{1}{2} \frac{GMm}{R_0} \frac{z^2}{R_0^2} + \frac{1}{6} \frac{GMm}{R_0} \frac{z^3}{R_0^3} - \dots$
 $U(R_0+z) = -\frac{GMm}{R_0} + \frac{GMm}{R_0} \frac{z}{R_0} - \frac{1}{2} \frac{GMm}{R_0} \frac{z^2}{R_0^2} + \frac{1}{6} \frac{GMm}{R_0} \frac{z^3}{R_0^3} - \dots$

$\vec{F} = -\nabla U$
 $m \ddot{\vec{r}} = \vec{F}$
 $\vec{v} = \dot{\vec{r}}$
 $\vec{v} \cdot \vec{v} = \dot{\vec{r}} \cdot \dot{\vec{r}} = \frac{d}{dt} \left(\frac{1}{2} m \dot{\vec{r}} \cdot \dot{\vec{r}} \right) = \dot{T}$
 $\vec{v} \cdot \vec{v} = \dot{\vec{r}} \cdot \dot{\vec{r}} = \frac{d}{dt} \left(\frac{1}{2} m \dot{\vec{r}} \cdot \dot{\vec{r}} \right) = \dot{T}$
 $\vec{v} \cdot \vec{v} = \dot{\vec{r}} \cdot \dot{\vec{r}} = \frac{d}{dt} \left(\frac{1}{2} m \dot{\vec{r}} \cdot \dot{\vec{r}} \right) = \dot{T}$

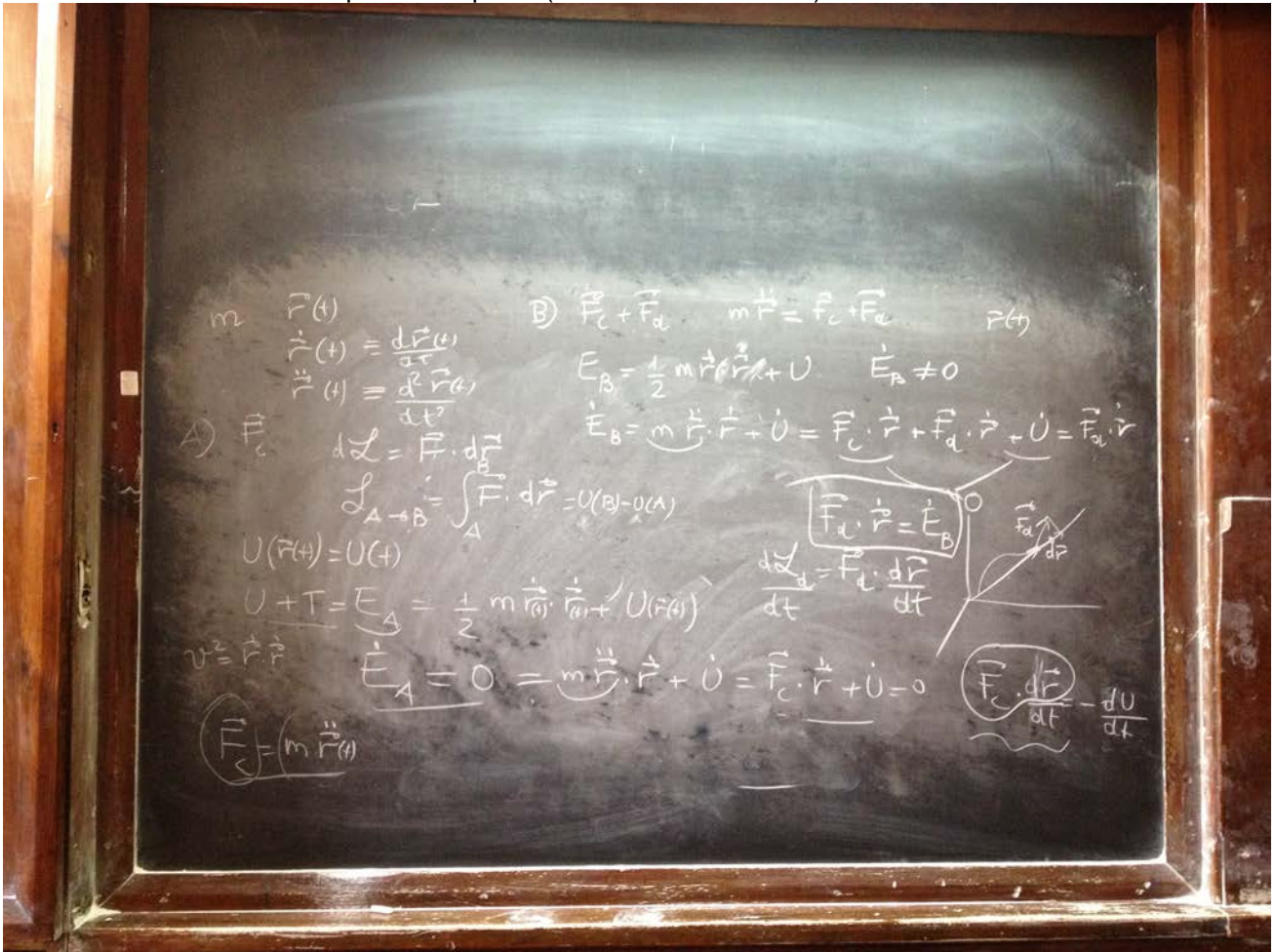
$E_A = U + T$
 $\dot{E}_A = \dot{U} + \dot{T} = 0$
 $E_A = U + T$
 $\dot{E}_A = \dot{U} + \dot{T} = 0$
 $E_A = U + T$
 $\dot{E}_A = \dot{U} + \dot{T} = 0$

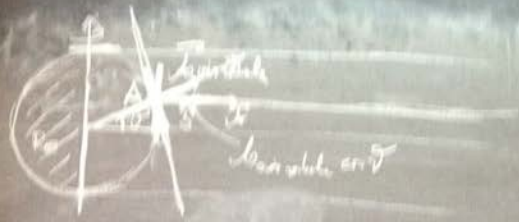
18 Febbraio 2014

Corpo soggetto solo a forza conservativa oppure a forza conservativa più dissipativa. Calcolo del lavoro fatto nell'unità di tempo dalla forza conservativa.

Calcolo del lavoro fatto nell'unità di tempo dalla forza dissipativa.

Di nuovo sulla soluzione del primo compito (del 12 dicembre 2013)





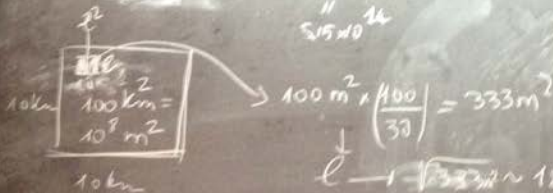
$P_{\text{inc}} = 100 \frac{\text{J}}{\text{cm}^2}$
 $C_{\text{O}} = 1.99 \times 10^3 \frac{\text{V}}{\text{m}^2}$
 $L = \text{m}^2$
 $I = L \cdot A$

$P_{\text{irradiata}} = C_{\text{O}} \cdot L \cdot \sin^2 \theta$
 $E = 15\% = \frac{15}{100} = 0.15$
 $I_{\parallel S} \quad I_{\perp S}$



$E_{\text{emessa per metro quadrato}} = C_{\text{O}} \cdot \epsilon \cdot \frac{16 \times 10^7 \text{ s}}{4\pi} \quad J = E \cdot 1.1 \times 10^{10} \text{ J}$
 $4\pi R_{\text{O}}^2 = 4\pi (6.37 \times 10^6 \text{ m})^2 = 5.1 \times 10^{14} \text{ m}^2$

$E_{\text{emessa}} \approx E \cdot 1.1 \times 10^{10} \text{ J} \approx 5.9 \times 10^{18} \text{ J}$



$l = 10 \text{ m} \quad l^2 = 10^2 \text{ m}^2$

$\frac{10^2 \text{ m}^2}{10^8 \text{ m}^2} = 10^{-6}$

$E_{\text{emessa}} \rightarrow \frac{5.2 \times 10^{18}}{6.7 \times 10^{20}} \approx 1.8 \times 10^{-3}$
 $E_{\text{tot } 20\%} \rightarrow \frac{1.8 \times 10^{-3}}{2 \times 10^{-5}} \approx 90$
 0.2%

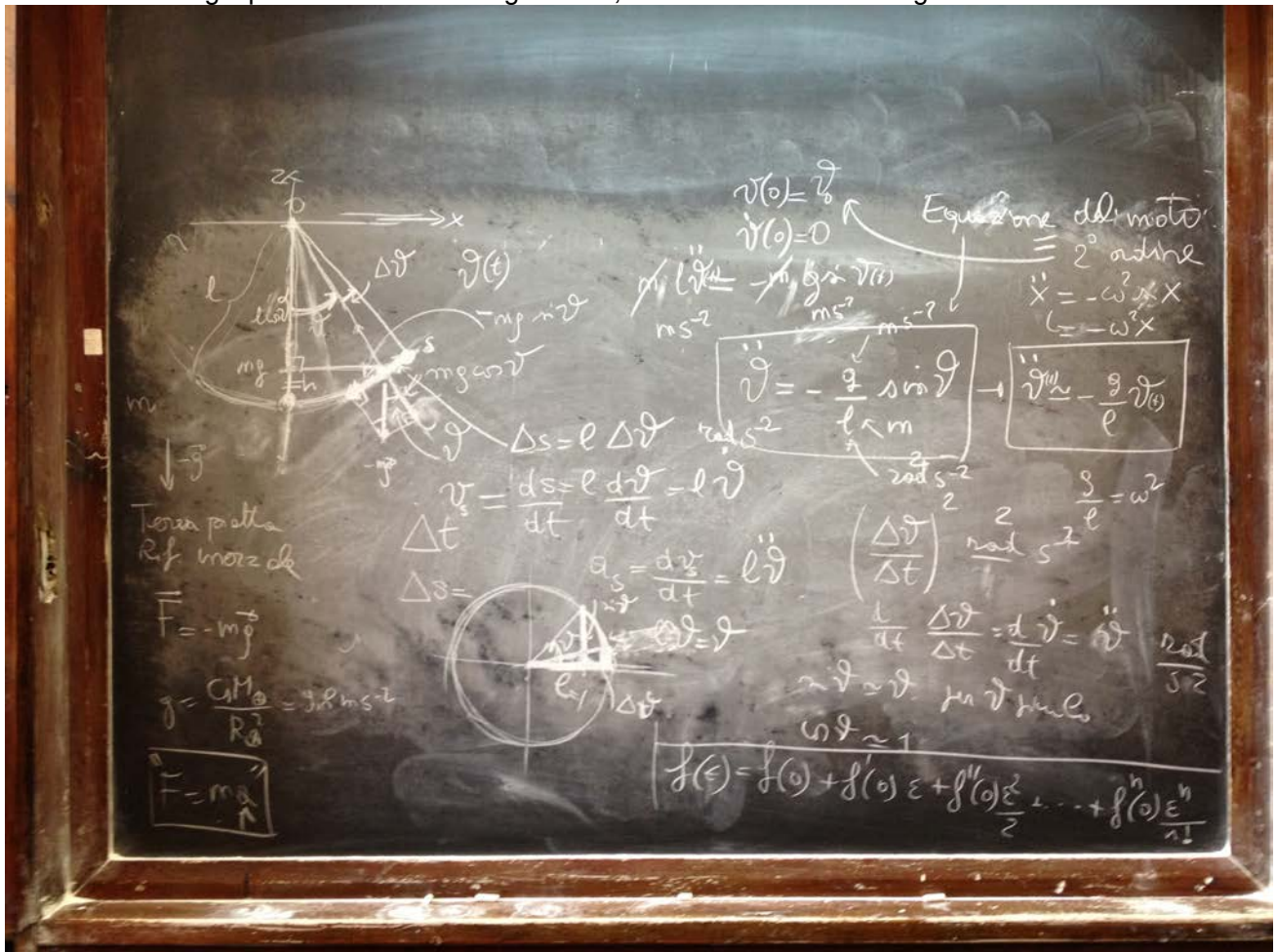
$P_{\text{tot}} = 1.5 \times 10^{13} \frac{\text{J}}{\text{s}}$
 $E_{\text{tot}} = 1.5 \times 10^{13} \cdot 3.16 \times 10^7 \text{ J}$
 $= 4.7 \times 10^{20} \text{ J}$

20 Febbraio 2014

Equazione del moto del pendolo semplice

Soluzione nel caso delle piccole oscillazioni (richiamo sulla espansione in serie di Taylor)

Calcolo dell'energia potenziale e dell'energia totale, conservazione dell'energia.



$$\omega = \frac{v_{max}}{l} = \frac{v_0}{l}$$

$$\begin{cases} x(t) = \cos(\omega t) \\ \dot{x}(t) = -\sin(\omega t) \cdot \omega \\ \ddot{x} = -\cos(\omega t) \cdot \omega^2 \\ \ddot{x} = -\frac{g}{l} x \\ y(t) = \sin(\omega t) \\ \dot{y} = \cos(\omega t) \cdot \omega \\ \ddot{y} = -\sin(\omega t) \cdot \omega^2 \\ \ddot{y} = -\frac{g}{l} y \end{cases}$$

$$x^2 + y^2 = l^2$$

$$\begin{aligned} \vartheta(t) &= A \cos \omega t + B \sin \omega t & \vartheta(0) &= \vartheta_0, \quad \dot{\vartheta}(0) = 0 \\ \vartheta(0) &= A = \vartheta_0 \\ \dot{\vartheta}(t) &= -A\omega \sin \omega t + B\omega \cos \omega t & \dot{\vartheta}(0) &= B\omega = 0 \\ \vartheta(t) &= \vartheta_0 \cos \omega t & \dot{\vartheta}(0) &= 0 \end{aligned}$$

$$\omega = \sqrt{\frac{g}{l}} \quad T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{l}{g}}$$

$U(0) - U(\vartheta) = - \int_{\vartheta_0=0}^{\vartheta} \vec{F} \cdot d\vec{s}$ Forza conservativa
 $L_{0 \rightarrow \vartheta} = \int_0^{\vartheta} mgl \sin \vartheta' d\vartheta' = -mgl \int_0^{\vartheta} \cos \vartheta' d\vartheta' = -mgl(\sin \vartheta - 0) = mgl(1 - \cos \vartheta)$
 $dL = \vec{F} \cdot d\vec{s} = -mgl \sin \vartheta$
 $U(\vartheta) = mgl(1 - \cos \vartheta)$
 $E = \frac{1}{2} m l^2 \dot{\vartheta}^2 + mgl(1 - \cos \vartheta) = \text{costante}$
 $\frac{d}{dt} E = 0$
 $m l^2 \dot{\vartheta} \ddot{\vartheta} + mgl \sin \vartheta = 0$
 $(\dot{\vartheta}) \ddot{\vartheta} = - \frac{g}{l} \sin \vartheta$
 $m s^{-1} \quad \ddot{\vartheta} = - \frac{g}{l} \sin \vartheta$
 $mgl(1 - \cos \vartheta_0) = \frac{1}{2} m v_{max}^2$
 $g l (1 - \cos \vartheta_0) = \frac{1}{2} v_{max}^2$
 $v_{max} = \sqrt{2gl(1 - \cos \vartheta_0)}$

25 Febbraio 2014

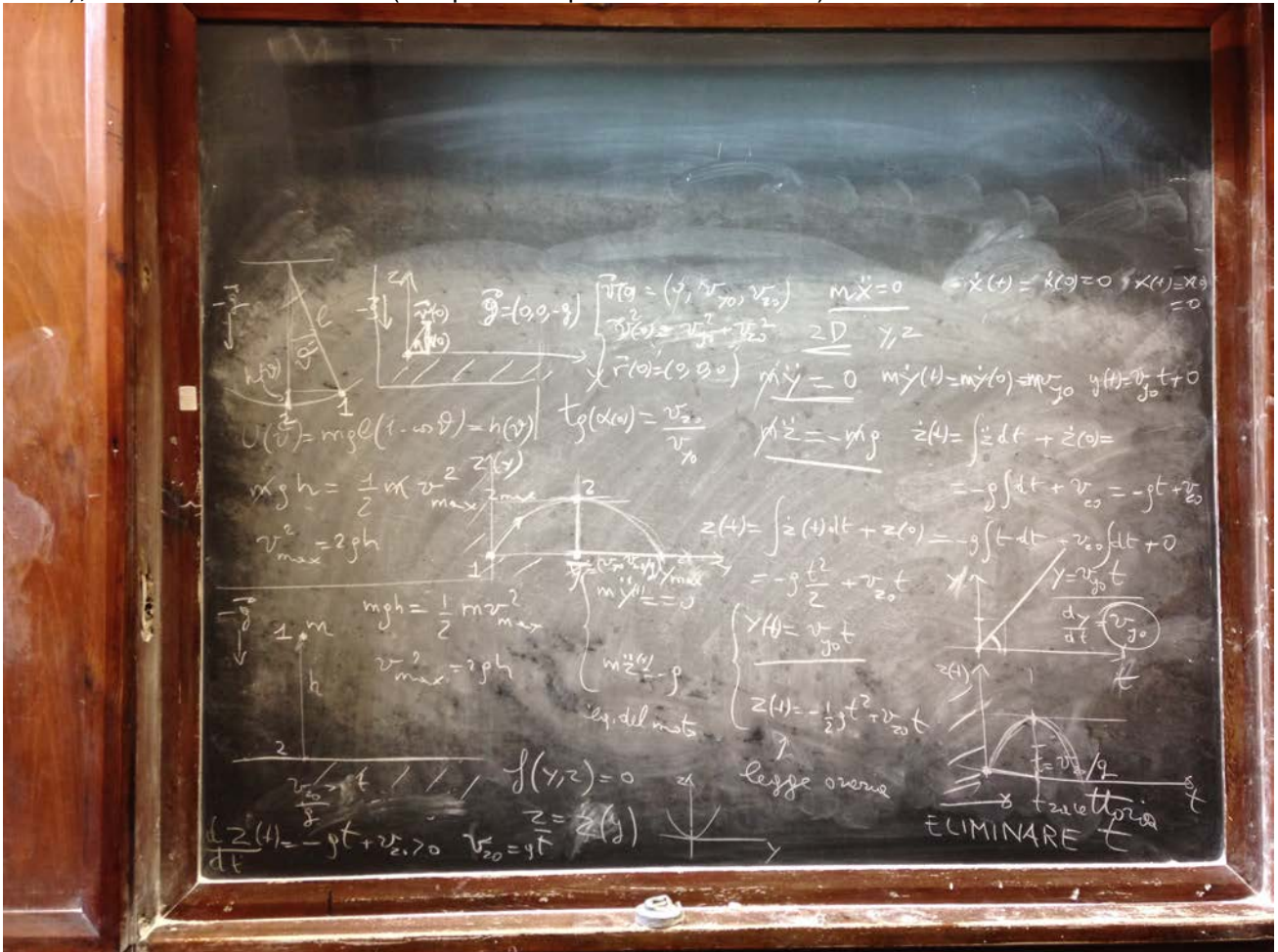
Conservazione energia nel caso del pendolo e del corpo in caduta

Moto del proiettile (terra piatta non rotante; riferimento inerziale): soluzione completa del problema (gradi di libertà; equazioni del moto; loro integrazione e legge oraria; equazione della traiettoria; uso della conservazione dell'energia)

Equazione di Newton e conservazione della quantità di moto lineare: problema del bambino sul lago ghiacciato

Riferimento in moto rettilineo uniforme rispetto ad un RI

Riferimento accelerato: eq del moto in un riferimento accelerato (Es: ascensore di Einstein in caduta libera); riferimento non inerziale (nel quale compaiono forze inerziali)



$$t = \frac{y}{v_{y0}}$$

$$z = -\frac{1}{2}g \frac{y^2}{v_{y0}^2} + \frac{v_{z0}}{v_{y0}} y$$

$$z = -\left(\frac{g}{2v_{y0}^2}\right)y^2 + \frac{v_{z0}}{v_{y0}} y$$

$$z_{max} = z(y) = -\frac{g}{2} \frac{v_{z0}^2}{v_{y0}^2} + \frac{v_{z0}^2}{v_{y0}^2} \frac{v_{y0}}{g} =$$

$$= -\frac{1}{2} \frac{v_{z0}^2}{g} + \frac{v_{z0}^2}{g} = \frac{v_{z0}^2}{2g}$$

$$v_{z0}^2 = 2gz_{max}$$

$$E_1 = \frac{1}{2} m (v_{y0}^2 + v_{z0}^2)$$

$$E_2 = \frac{1}{2} m v_{y0}^2 + mgz_{max}$$

$$\frac{dz}{dy} = -\frac{g}{v_{y0}^2} y + \frac{v_{z0}}{v_{y0}}$$

$$-\frac{g}{v_{y0}^2} y = -\frac{v_{z0}}{v_{y0}} \Rightarrow y = \frac{v_{z0} v_{y0}}{g}$$

$$y_{max} = 2y = \frac{2v_{z0} v_{y0}}{g}$$

$$z_{max} = \frac{g}{2g} \left(\frac{2v_{z0} v_{y0}}{g}\right)^2 + \frac{v_{z0}}{v_{y0}} \frac{2v_{z0} v_{y0}}{g} = 0$$

$$y_{max} = \frac{2v_{z0} v_{y0}}{g}$$

$F = m\ddot{r}$ Riferimento inerziale

$$= m \frac{d\dot{r}}{dt} = \frac{d}{dt}(m\dot{r}) = \dot{p}$$

$\vec{p} = m\dot{r}$ quantità di moto lineare

$$\text{Se } F=0 \Rightarrow \dot{p} = \text{costante}$$

Diagram showing a mass m on a surface with forces M and $m < M$. A coordinate system (x, y) is shown with a point P moving along a path.

Equations for position and velocity in two frames:

$$x = x' + vt \quad x = \dot{x}' + vt$$

$$y = y' \quad \dot{y} = \dot{y}'$$

Force equations:

$$F = m\ddot{r}$$

$$\vec{F} = (M+m)\ddot{\vec{r}} = 0$$

$$\frac{d}{dt}[(M+m)\dot{\vec{r}}] = 0$$

$$(M+m)\dot{\vec{r}}(0) = m\dot{\vec{r}}_{iniz} + M\dot{\vec{r}}_{banch}$$

$$\vec{p}(0) = \vec{p}(1)$$

Velocity components:

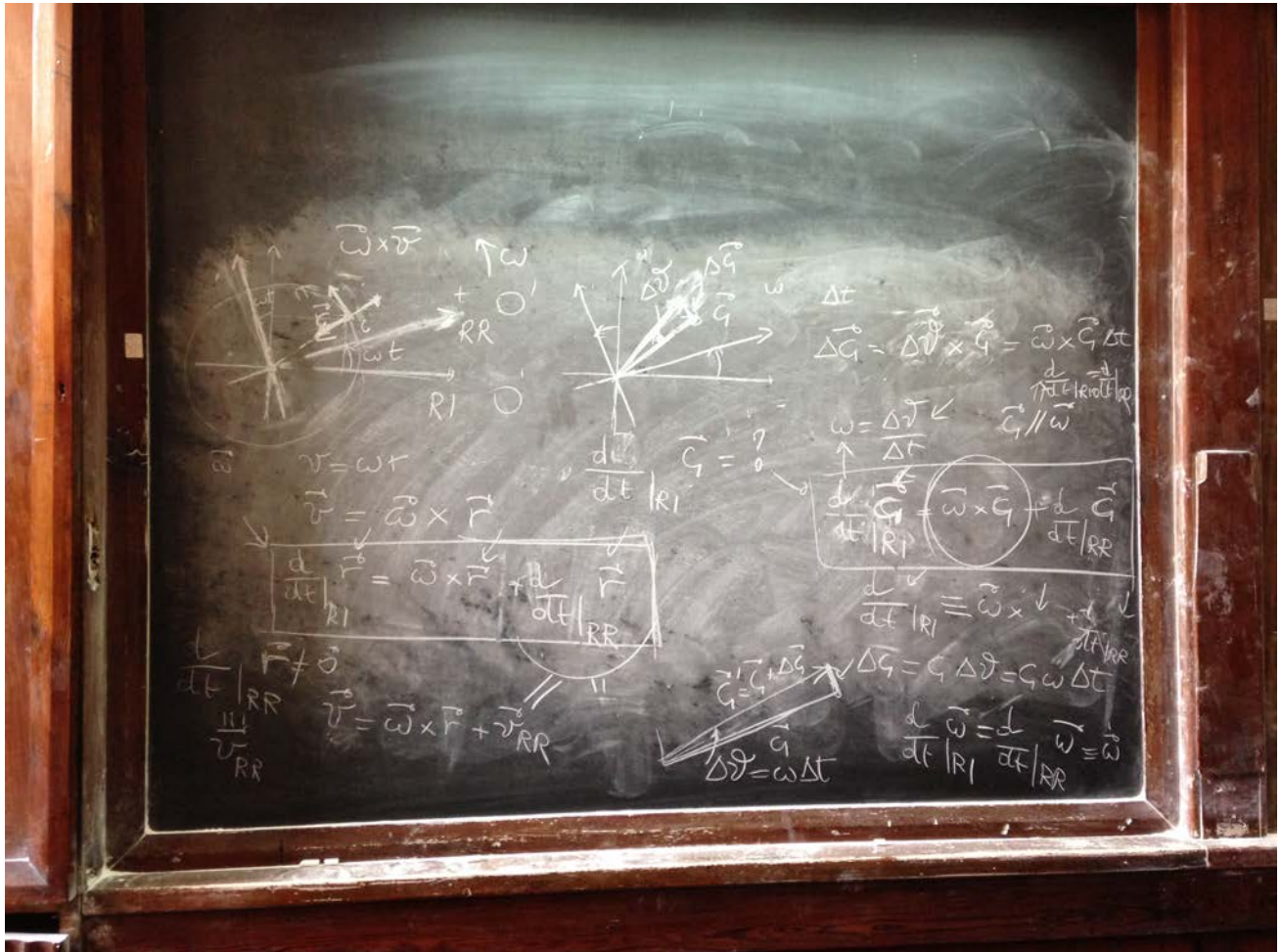
$$\dot{r}_{iniz} = -\frac{m}{M} \frac{v}{m}$$

$$m\dot{r}_{iniz} = -\frac{m^2}{M} \frac{v}{m}$$

$$m\dot{r}_{iniz} = -\frac{F}{M} \frac{m}{g}$$

27 Febbraio 2014

Un riferimento rotante è non inerziale anche se ruota a velocità angolare costante. Facciamo i conti nel piano perpendicolare al vettore velocità angolare di rotazione. Uso l'esempio della giostra, del vettore posizione e del vettore velocità del cavalluccio; dei vettori posizione e velocità del bambino che corre sulla giostra. La formula importante è quella che lega la derivata temporale di un vettore in un riferimento inerziale a quella in un riferimento rotante. Usando questa, arrivo alla formula generale che fornisce l'equazione del moto in un riferimento rotante (non inerziale), con la forma specifica di tutte le forze aggiuntive inerziali (in quanto proporzionali alla massa inerziale) che compaiono in questo sistema di riferimento.



$$t = \frac{y}{v_{y0}}$$

$$z = -\frac{1}{2}g \frac{y^2}{v_{y0}^2} + \frac{v_{z0}}{v_{y0}} y$$

$$z = -\left(\frac{g}{2v_{y0}^2}\right) y^2 + \frac{v_{z0}}{v_{y0}} y$$

$$z_{max} = z(y) = -\frac{g}{2} \frac{v_{z0}^2}{v_{y0}^2} + \frac{v_{z0}^2}{v_{y0}^2} \frac{v_{y0}}{g} =$$

$$= -\frac{1}{2} \frac{v_{z0}^2}{g} + \frac{v_{z0}^2}{g} = \frac{v_{z0}^2}{2g}$$

$v_{z0}^2 = 2gz_{max}$

$$E_1 = \frac{1}{2} m(v_{x0}^2 + v_{z0}^2)$$

$$E_2 = \frac{1}{2} m v_{y0}^2 + m g z_{max}$$

$$\frac{dz}{dy} = -\frac{g}{v_{y0}^2} y + \frac{v_{z0}}{v_{y0}}$$

$$-\frac{g}{v_{y0}^2} y = -\frac{v_{z0}}{v_{y0}} \Rightarrow y = \frac{v_{z0} v_{y0}}{g}$$

$$y_{max} = 2y = \frac{2v_{z0} v_{y0}}{g}$$

$$z_{max} = \frac{g}{2g} \left(\frac{v_{z0} v_{y0}}{g}\right)^2 + \frac{v_{z0}}{v_{y0}} \frac{v_{z0} v_{y0}}{g} = 0$$

$$y_{max} = 2 \frac{v_{z0} v_{y0}}{g}$$

$F = m\ddot{r}$ Riferimento inerziale

$$= m \frac{d\dot{r}}{dt} = \frac{d}{dt}(m\dot{r}) = \dot{p}$$

$\vec{p} = m\dot{r}$ quantità di moto lineare

$\sum F = 0 \Rightarrow \dot{p} = \text{costante}$

$\vec{F} = (M+m)\frac{d\vec{v}}{dt} = 0$

$\frac{d}{dt}[(M+m)\vec{v}] = 0$

$(M+m)\vec{v}(0) = m\vec{v}_{nervo} + M\vec{v}_{bambino}$

$\vec{v}(0) = \vec{v}(1)$

$\vec{v}_{bamb} = -\frac{m}{M} \vec{v}_{nervo}$

$\vec{p} = \vec{p}' + m\vec{a}$

$m\vec{v}' = m\vec{v} - m\vec{a}$

$m\vec{v}' = \vec{F} - m\vec{a}$

autobus

1	2	3
1	2	3
1	2	3

$x = x' + vt$ $x = \dot{x}' + vt$ $\ddot{x} = \ddot{x}'$
 $y = y'$ $\dot{y} = \dot{y}'$ $\ddot{y} = \ddot{y}'$

$\vec{a} \neq 0$ \vec{F}, \vec{F}'

$\vec{F}' = \vec{F} - m\vec{a}$

$m\vec{v}' = \vec{F} - m\vec{a}$

4 Marzo 2014

Moto del pendolo in presenza di aria: forza dissipativa proporzionale alla velocità e soluzione della equazione del pendolo in presenza di questa forza aggiuntiva

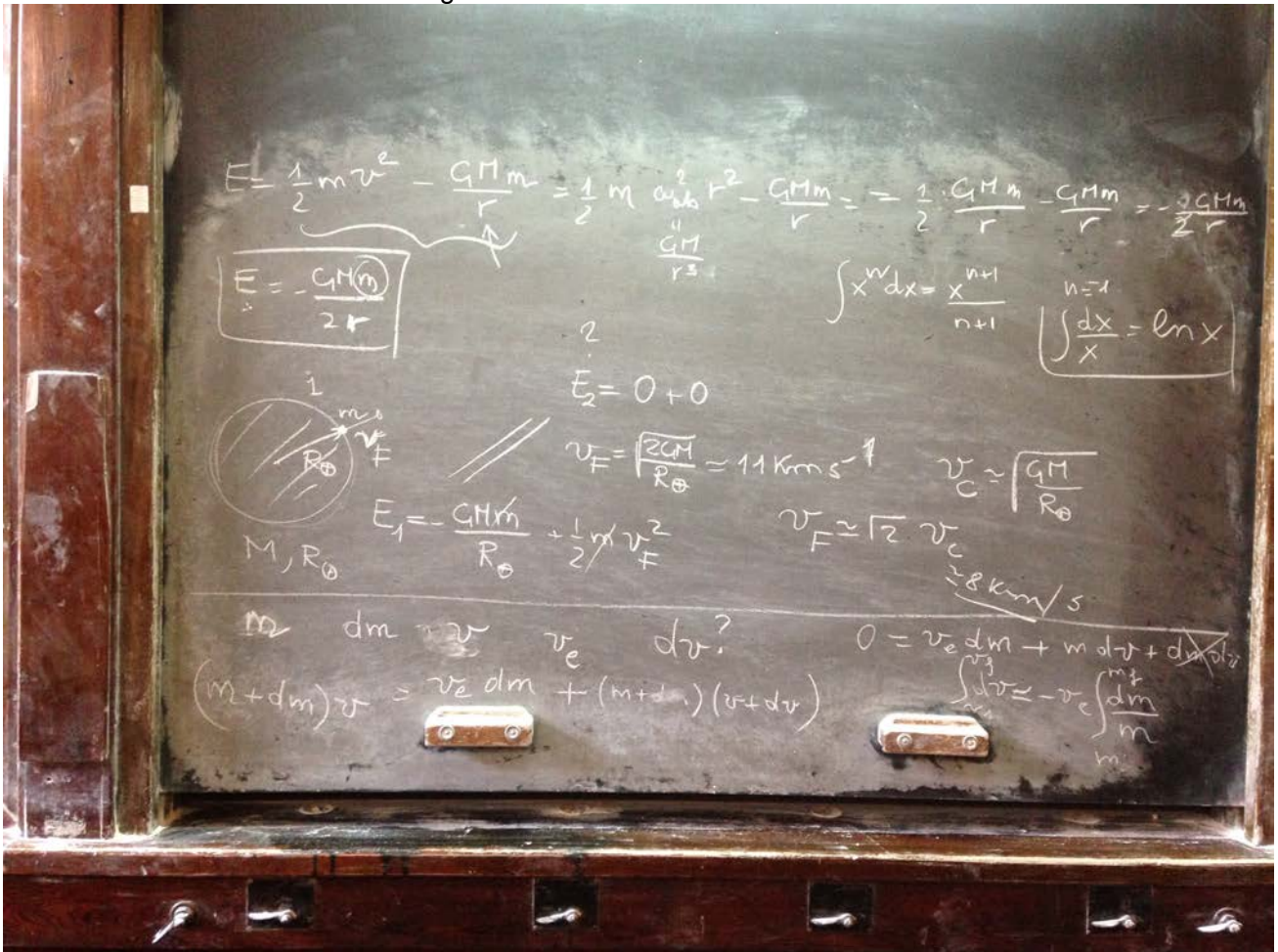
11 Marzo 2014

Riprendo il caso della forza di Coriolis nel caso dei venti e del loro effetto sugli aerei che volano ad alta quota.

Uso la formula delle equazioni del moto in un riferimento rotante applicandola ad un satellite artificiale in orbita circolare attorno alla Terra. Faccio il caso semplice di un satellite di massa trascurabile rispetto alla Terra, che quindi posso considerare fissa nell'origine. Ne ottengo la terza legge di Keplero. Usando poi la relazione tra velocità lineare e velocità angolare trovo la formula che fornisce la velocità lineare in funzione del raggio dell'orbita.

Scrivo la formula dell'energia totale del satellite (cinetica più potenziale) e calcolo il suo valore (costante del moto), che dipende solo dal raggio dell'orbita

Problema del razzo e richiamo sui logaritmi in base e





$$\vec{\omega} = \omega \hat{z} = \frac{2\pi}{86164} \text{ rad/s}$$

$$R_{\oplus} \approx 6.4 \times 10^6 \text{ m}$$

$$M_{\oplus} = 5.97 \times 10^{24} \text{ kg}$$

$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ s}^{-2} \text{ kg}^{-1}$$

$$r \approx R_{\oplus}$$

$$[\mu] = \text{km}^3 \text{ s}^{-2}$$

$$\omega_{orb} = \frac{2\pi}{P_{orb}}$$

$$P_{orb} = \frac{2\pi}{\omega_{orb}} = 80' \quad \omega_{orb}^2 = \frac{4 \times 10^4}{(6.4 \times 10^6)^2}$$

$$G M_{\oplus} = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} = 4 \times 10^{14} \text{ m}^3 \text{ s}^{-2}$$

$$\omega_{orb}^2 r^3 = G M_{\oplus}$$

$$m \vec{a}_{RR} = m \vec{a}_{RI} - 2m \vec{\omega} \times \vec{v}_{RR} - m \vec{\omega} \times \vec{r} - m \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

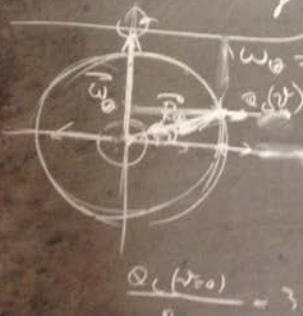
$$m \vec{a}_{RI} = \vec{F} = -\frac{GMm}{r^2} \hat{r}$$

$$m \vec{a}_{RR} = -\frac{GMm}{r^2} + m \omega^2 r$$

$$m \ll M \quad \vec{r} = \omega t \hat{r} = \frac{GM}{\omega^2} \hat{r}$$

$$v_2 - v_1 = -v_e \ln \left[\frac{m_f}{m_i} \right] = -v_e (\ln m_f - \ln m_i) = -v_e \ln \frac{m_f}{m_i}$$

$$\Delta v = v_e \ln \frac{m_i}{m_f}$$



$$\omega_0 = 7.3 \times 10^{-5} \text{ rad/s}$$

$$|\vec{\omega}_0 \times \vec{R}_0| = \omega_0 R_0 \sin \theta$$

$$\vec{\omega}_0 \times (\vec{\omega}_0 \times \vec{R}_0)$$

$$a_c(\theta) = \omega_0^2 R_0 \sin^2 \theta$$

$$a_c(90^\circ) = \omega_0^2 R_0 = 3.6 \times 10^{-3} \text{ m/s}^2$$

$$e^x = y$$

$$\ln e^x = \ln y$$

$$x = \ln y$$

$$e^0 = b$$

$$a = \ln b$$

$$e^a = yb$$

$$x + a = \ln(yb)$$

$$\int v_e dm = 2000 \text{ kg}$$

$$dm \ll m \quad dm \ll r$$

13 Marzo 2014

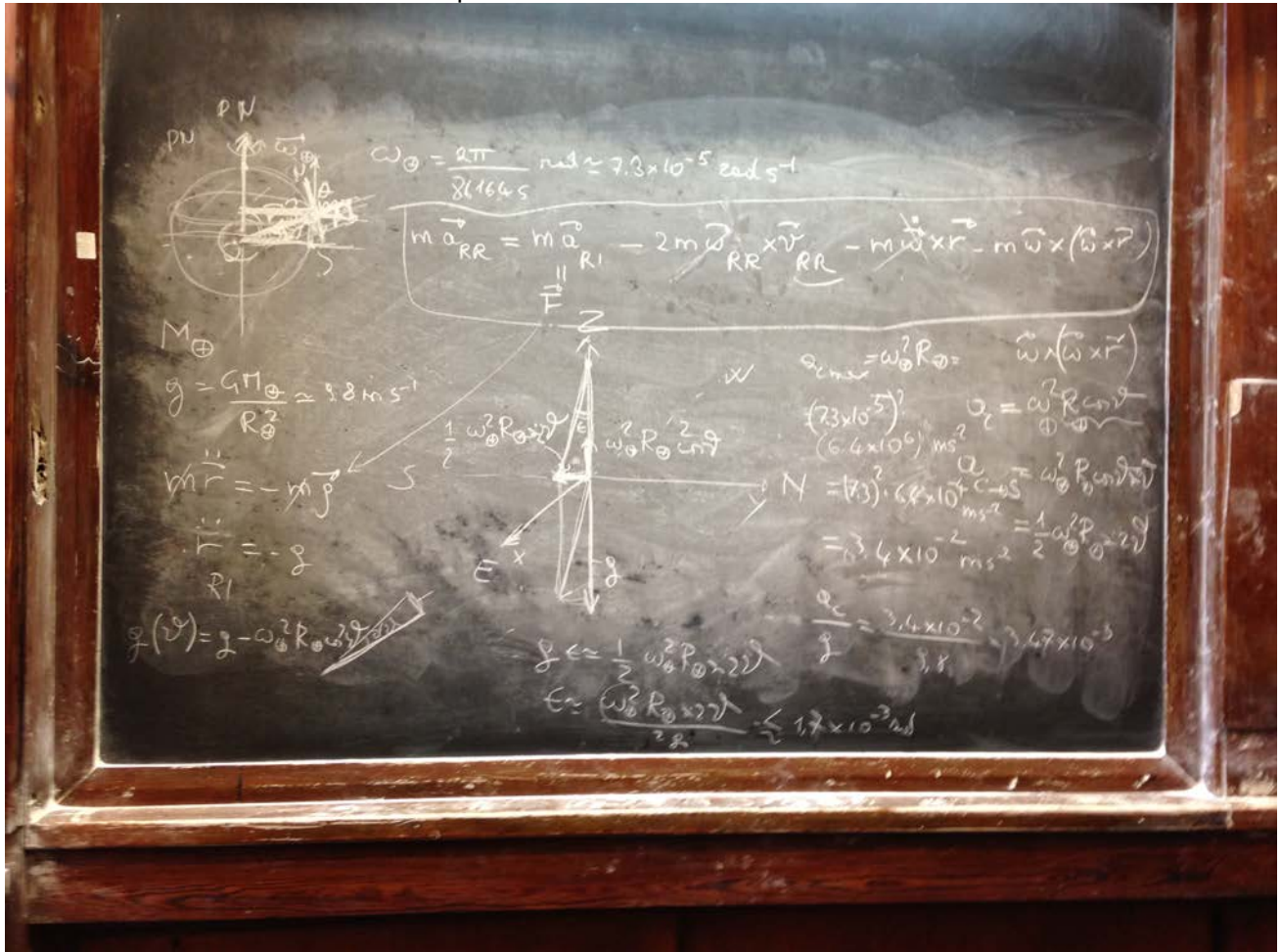
Accelerazione centrifuga sulla Terra in rotazione. Rapporto numerico con l'accelerazione gravitazionale all'equatore

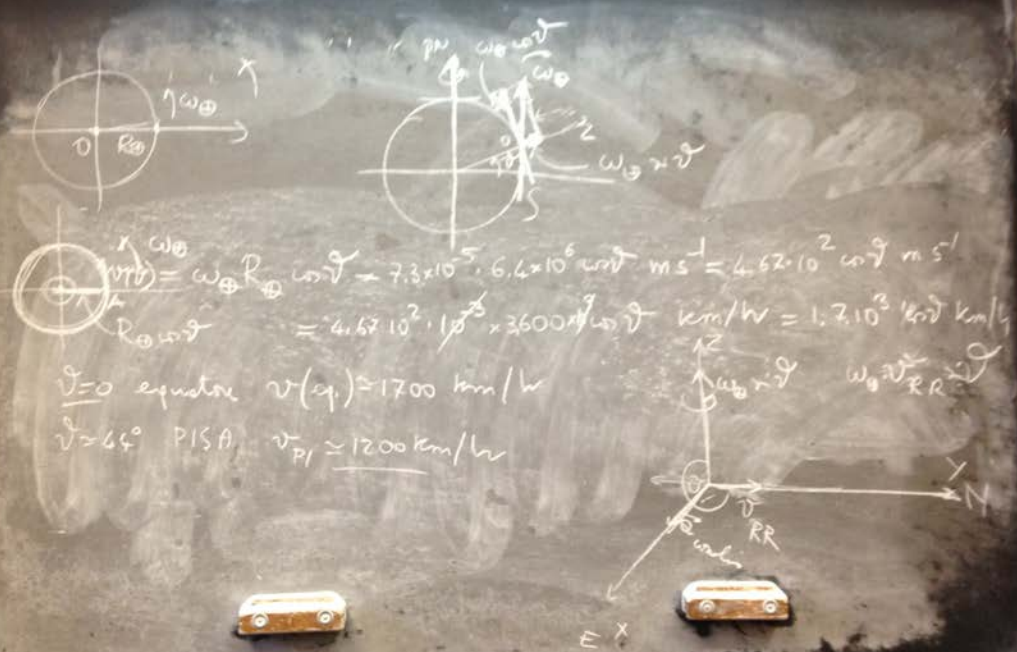
Effetti sulla accelerazione di gravità locale in funzione della latitudine dell'osservatore

Velocità di rotazione rispetto al sistema inerziale ("stelle fisse") di un osservatore sulla Terra ad una data latitudine

Moto del proiettile sulla Terra rotante; effetto delle accelerazione di Coriolis sul moto del proiettile e sua deviazione

Effetto delle accelerazione di Coriolis per una massa in caduta libera ad una data latitudine





$$v_{\text{earth}} = \omega_{\text{earth}} R_{\text{earth}} = 7.3 \times 10^{-5} \cdot 6.4 \times 10^6 \text{ m/s} = 4.67 \cdot 10^2 \text{ m/s}$$

$$= 4.67 \cdot 10^2 \cdot 3600 \text{ km/h} = 1.7 \cdot 10^3 \text{ km/h}$$

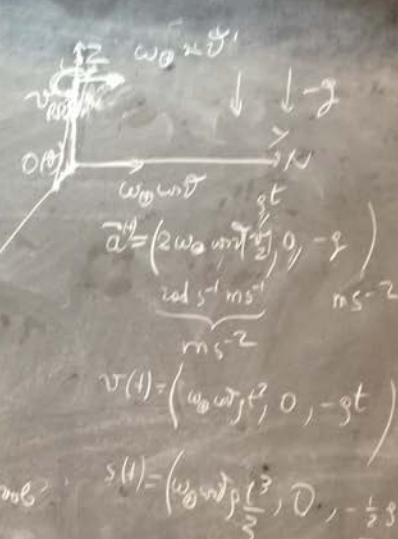
$\vartheta = 0$ equator $v(\vartheta) = 1700 \text{ km/h}$
 $\vartheta = 45^\circ$ PISA $v_{\text{PI}} = 1200 \text{ km/h}$

$$\vec{a}_{\text{centrifugal}} = (2\omega_{\text{earth}} v_{\text{RR}} + \dot{v}, 0, 0)$$

$$\vec{v}(t) = (2\omega_{\text{earth}} v_0 + \dot{v} \cdot t, v_0, 0)$$

$$\vec{z}(t) = \left(\omega_{\text{earth}} v_0 \cdot 2t + \frac{v_0 t^2}{v_0}, v_0 t, 0 \right)$$

$$\frac{v_0 t^2}{v_0} = \frac{v_0 v_0 t^2}{v_0 t} = (v_0 + \dot{v}) t_{\text{mlb}}$$



$$\vec{a} = (2\omega_{\text{earth}} v_0, 0, -g)$$

$$v(t) = (\omega_{\text{earth}} v_0 t^2, 0, -gt)$$

$$s(t) = \left(\omega_{\text{earth}} v_0 \frac{t^3}{3}, 0, -\frac{1}{2}gt^2 + h \right)$$

K_{11}
 $t=0$ h
 $v/d=0$
 34

$$0 = h - \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2h}{g}}$$

18 Marzo 2014

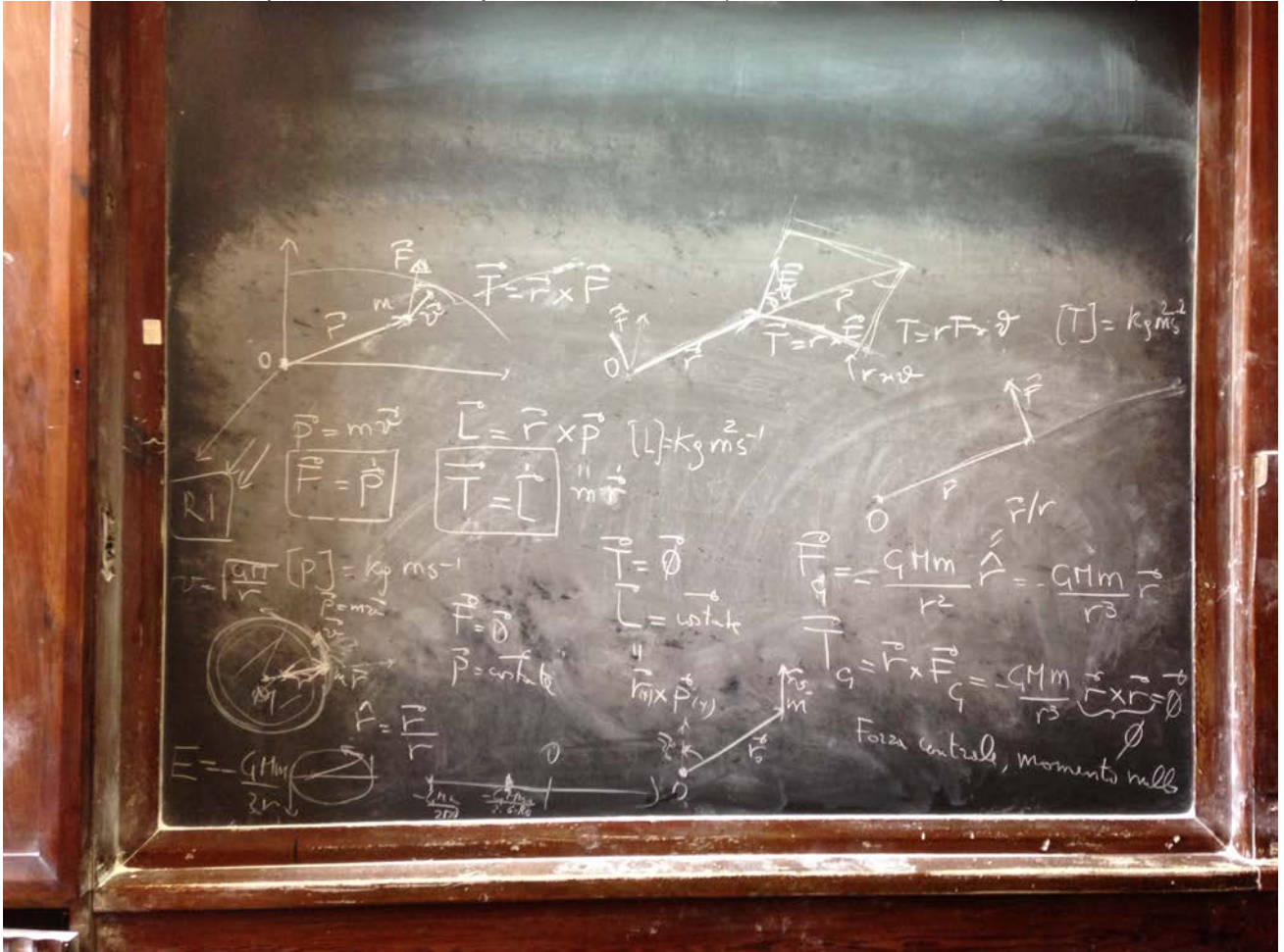
Momento angolare e momento di una forza. Definizioni ed equazione fondamentale di Newton

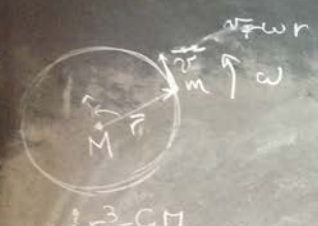
Il caso delle forze centrali (conservazione del momento angolare)

Orbita di un satellite: conservazione del momento angolare, velocità areolare e seconda legge di Keplero


Problema del pendolo semplice risolto usando momento angolare e momento della forza gravitazionale

Conservazione della quantità di moto e problema del razzo (risistemato dalla volta precedente)






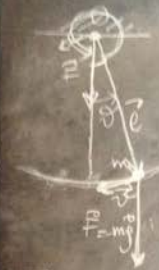
$\omega r = v$
 $\omega^2 r = GM/r^2$
 $\vec{L} = m \vec{r} \times \vec{v} = \text{constate}$
 $L = m r v = m r^2 \omega = \text{constate}$



$\frac{dv}{dt} = \frac{1}{2} r \frac{d^2 \theta}{dt^2} = \frac{1}{2} r \frac{d^2 \varphi}{dt^2}$
 $= \omega \frac{dv}{dt}$



$\vec{r} = r \hat{e}_r + r \dot{\theta} \hat{e}_\theta$
 $\hat{e}_r \times \vec{r} = 0$
 $L = m r^2 \dot{\theta} = \text{constate}$



$m(t) \quad v(t) \quad r(t) \quad dv \quad dr$
 $m \dot{v} = (m+dm)(v+dv) + (v+dv)dm$
 $= mv + m dv + v dm + dm dv + v^2 \frac{dm}{m} = mv + m dv + v dm + v^2 \frac{dm}{m}$
 $m dv = -v^2 \frac{dm}{m}$

$\vec{T} = \vec{l} \times \vec{F} = -m \vec{l} \times \hat{j}$
 $\vec{L} = m \vec{l} \times \vec{v}$
 $v = l \dot{\theta}$

\perp piawa luraña entzante $T = mgl \sin \theta$
 \perp piawa luraña entzante $L = m l v = m l^2 \dot{\theta}$

$\vec{r}(t) = (0, y, h)$
 $\dot{\vec{r}}(t) = (0, \dot{y}, 0)$
 $\vec{r}(t) = (l \cos \theta, l \sin \theta, 0)$
 $\dot{\vec{r}}(t) = (-l \dot{\theta} \sin \theta, l \dot{\theta} \cos \theta, 0)$

$\ln \frac{m}{m_0} = \frac{1}{v} \int v dv$
 $\frac{m}{m_0} = e^{-\frac{v^2}{2g}}$

20 Marzo 2014

Caduta dei gravi, accelerazione di Coriolis, equazioni del moto. Loro integrazione. Equazione della traiettoria

Forza elastica, equazione del moto (oscillatore armonico), frequenza angolare (o pulsazione), forma generale della soluzione.

Caso di assenza di perdite: forza elastica conservativa, calcolo della energia potenziale

The chalkboard contains the following content:

- Top Left Diagram:** A circular diagram representing Earth's rotation with angular velocity $\vec{\omega}_\oplus$ and a point S on the surface.
- Top Middle Diagram:** A 3D coordinate system with axes x, y, z . A mass m is shown falling from height h . The Earth's angular velocity is $\omega_\oplus = 7.3 \times 10^{-5} \text{ rad/s}$. The Coriolis acceleration is $\vec{a}_{Coriolis} = 2\vec{\omega}_\oplus \times \vec{v}$.
- Top Right Equations:**
 - $\vec{r}(0) = (0, 0, h)$
 - $\dot{\vec{r}}(0) = (0, 0, 0)$
 - $g \approx 9.8 \text{ m/s}^2$
 - $g = \frac{GM_\oplus}{R_\oplus^2}$
 - $h \ll R_\oplus$
- Middle Equations:**
 - $m \cdot \vec{a} = m \cdot \vec{a}_{RR} - 2m\vec{\omega}_\oplus \times \vec{v}_{RR} - m(\vec{\omega}_\oplus \times (\vec{\omega}_\oplus \times \vec{r}))$
 - $\vec{r}(t) = (\omega_\oplus \cos \theta \cdot g t^2, 0, -g t)$
 - $\vec{r}(t) = (\frac{1}{3} \omega_\oplus \cos \theta \cdot g t^3, 0, -\frac{1}{2} g t^2 + h)$
 - $\int t^2 dt = \frac{t^3}{3}$
 - $\text{rad/s}^3 \cdot \text{m/s}^2 \cdot \text{s}^3 = \text{m}$
 - $h = h = h^{3/2} = \frac{2h}{3}$
 - 40 m
- Bottom Left Diagram:** A circular diagram showing the path of a falling object on a rotating Earth, with Earth's mass M_\oplus and radius R_\oplus .
- Bottom Equations:**
 - $a_{Coriolis} = 2\omega_\oplus \cos \theta \cdot v(t)$
 - $t_{vol} = \sqrt{\frac{2h}{g}}$
 - $\vec{r}(t_{vol}) = (\frac{2}{3} \omega_\oplus \cos \theta \cdot \sqrt{\frac{2h}{g}}^3, 0, 0)$
 - $X_{Est} = \frac{2}{3} h \omega_\oplus \cos \theta \cdot \sqrt{\frac{2h}{g}} = 5.55 \times 10^{-3} \text{ (rad/s)} \cdot \text{m} \cdot \text{s} \cdot \text{m}^{-1/2} \cdot \text{s}^{1/2}$
 - $t_{vol} = \sqrt{\frac{2h}{g}} = 2.86 \text{ s}$
 - $X_{Est} \approx 4 \times 10^{-3} \text{ m}$

$$x^{2/3} = \frac{2}{3^{2/3}} \frac{\omega_0^{2/3} (\cos \omega t)^{2/3}}{g^{1/3}} (h-z)$$

$$z(t) = h - \frac{1}{2} g t^2 \quad \text{Est} \quad z = z(x)$$

$$x_{Est} = \frac{1}{3} g \omega_0 \cos \omega t \quad t^3$$

$$f(x, z) = 0$$

$$z = \frac{1}{2} g t^2 = h - z$$

$$t = \sqrt{\frac{2(h-z)}{g}}$$

$$x = \frac{1}{3} g \omega_0 \cos \omega t \quad z^{3/2} (h-z)^{3/2}$$

$$x = \frac{2^{3/2}}{3} \frac{\omega_0 \cos \omega t}{g^{3/2}} (h-z)^{3/2}$$

$$x^{2/3} = A(\theta) (h-z) \quad z = z(x)$$

$$\frac{x^{2/3}}{A(\theta)} = h-z \Rightarrow z = h - \frac{1}{A(\theta)} x^{2/3}$$


$$x=0, z=h$$

$$z=0 \quad h = \frac{1}{A(\theta)} x^{2/3} \quad \text{Est}$$


$$x_{Est}^{2/3} = A(\theta) h = \frac{2}{3^{2/3}} \frac{\omega_0^{2/3} (\cos \omega t)^{2/3}}{g^{1/3}} h$$

$$x_{Est} = \frac{2^{3/2}}{3} \frac{\omega_0 \cos \omega t}{g^{3/2}} h^{3/2}$$

$$= \frac{2}{3} h \omega_0 \cos \omega t \sqrt{\frac{2h^{1/2}}{g}}$$

$k \times 0$
 $F = -kx \quad [k] = N/m$

 $x=0 \quad F=0$ punto di equilibrio
 $F \propto -x$


 $F = -k(x-x_0)$
 $x' = x - x_0$

$U(x) = \int kx' dx = \frac{1}{2} kx'^2$
 $U(x) = \frac{1}{2} kx^2$

 $x=0$ posizione di equilibrio
 (potenziale a zero della molla)

$$\ddot{x} = -\frac{k}{m} x(t)$$

$$m\ddot{x} + kx = 0$$

$$\dot{x} + \frac{k}{m} x = 0$$

$$\ddot{x} + \omega^2 x = 0$$

$$x(0) = x_0$$

$$\dot{x}(0) = v_0$$

$$x(t) = C_1 \cos \omega t + C_2 \sin \omega t$$

$$x(t) = a \cos(\omega t + \alpha) = a \cos \omega(t - t_0)$$

$$E(x, \dot{x}) = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2(t)$$

$$\left[\frac{k}{m} \right] = \frac{N}{m} \frac{1}{kg \frac{m}{s^2}} = \frac{kg \cdot m \cdot s^{-2}}{kg \cdot m} = s^{-2}$$

$$\omega^2 = \frac{k}{m} \quad P = \frac{2\pi}{\omega} \quad (\omega) = \frac{rad}{s}$$

$$|\omega = 2\pi \nu| \quad \nu = \frac{1}{P} \text{ Hz}$$

$$1 \text{ MHz} = 10^6 \text{ Hz}$$

$$P = 10^{-6} \text{ s}$$

$$P = \frac{1}{2.9} 10^{-9} \text{ s}$$

28

25 Marzo 2014

Caso di due masse accoppiate da una forza elastica in una sola direzione. Moto relativo e riduzione ad un solo corpo (centro di massa, conservazione della quantità di moto e massa ridotta). Energia totale di un oscillatore armonico.

$F = -kx$ $[k] = N/m$ $\left[\frac{k}{m}\right] = \frac{kg \cdot s^{-2}}{kg} = s^{-2}$ T $\nu = \frac{1}{T}$
 S $s^{-1} \equiv Hz$
 $2\pi\nu = \omega$
 $[\omega] = rad \cdot s^{-1}$
 $\frac{2\pi}{T} = \omega$
 $\nu = 52.5 MHz = 5.25 \cdot 10^6 s^{-1}$
 $T = \frac{1}{\nu} = \frac{1}{5.25 \cdot 10^6} s = \frac{1}{5.25} \cdot 10^{-6} s$

Oscillatore armonico
 $m\ddot{x} = -kx$
 $\ddot{x} + \frac{k}{m}x = 0$ $\frac{k}{m} \equiv \omega^2$
 $\ddot{x} + \omega^2 x = 0$ $\omega = \sqrt{\frac{k}{m}}$

$x = x - x_0$
 $F = -k(x - x_0)$
 $U(x) = -\int F dx = -\int -k(x - x_0) dx = \frac{1}{2}k(x - x_0)^2$
 $x_0 = 0$
 $U(x) = \frac{1}{2}kx^2$

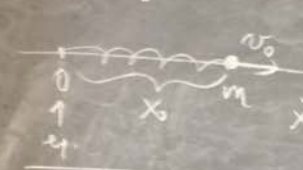
$x(t) = C_1 \cos \omega t + C_2 \sin \omega t$
 $x(t) = a \cos(\omega t + \alpha)$
 $x(0) = a \cos(\omega \cdot 0 + \alpha) = a \cos \alpha$
 $\dot{x}(0) = -a\omega \sin(\omega \cdot 0 + \alpha) = -a\omega \sin \alpha$
 $\alpha = \pi/2$ $\alpha = \pi$

$E(x, \dot{x}) = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2$
 $E = \frac{1}{2}m\omega^2 a^2 = \frac{1}{2}k a^2$
 $2\pi = \frac{C_1 \pi m}{2r}$

$x = -a \cos(\omega t)$
 $\dot{x} = a\omega \sin(\omega t)$
 $\omega^2 r^3 = GM$

$x(t) = a \cos(\omega t + \alpha)$
 $\omega = \sqrt{\frac{GM}{r^3}}$

$x(0) = x_0$ spostamento dall'equilibrio ($x=0$) all'istante $t=0$
 $v(0) = v_0$



$x(t) = a \cos(\omega t + \alpha)$ and $\alpha \leftrightarrow x_0, v_0$
 $x(0) = a \cos \alpha = x_0$
 $\dot{x} = \frac{dx(t)}{dt} = -a\omega \sin(\omega t + \alpha)$
 $\dot{x}(0) = -a\omega \sin \alpha = v_0$
 $\omega^2 = \frac{k}{m}$

$t_{sp} = -\frac{v_0}{\omega x_0}$
 $v^2 = x_0^2 + \frac{v_0^2}{\omega^2}$

$E(x, \dot{x}) = \frac{1}{2} m a^2 \omega^2 \sin^2(\omega t + \alpha)$
 $= \frac{1}{2} m a^2 \omega^2 \cos^2(\omega t + \alpha)$
 $= \frac{1}{2} m a^2 \omega^2 (\sin^2 + \cos^2) = \frac{1}{2} m a^2 \omega^2$

$x_0 = a \cos \alpha$
 $v_0 = -a\omega \sin \alpha$
 $\cos \alpha = \frac{x_0}{a}$
 $\sin \alpha = -\frac{v_0}{a\omega}$

$x(t) = c_1 \cos \omega t + c_2 \sin \omega t$
 $x(t) = a \cos(\omega t + \alpha) = a \cos \alpha \cos \omega t - a \sin \alpha \sin \omega t = (a \cos \alpha) \cos \omega t + (-a \sin \alpha) \sin \omega t$
 $x(t) = a \cos \omega(t - t_0)$
 $\alpha = -\omega t_0$
 $c_1 = a \cos \alpha$
 $c_2 = -a \sin \alpha$
 $c_1^2 + c_2^2 = a^2$

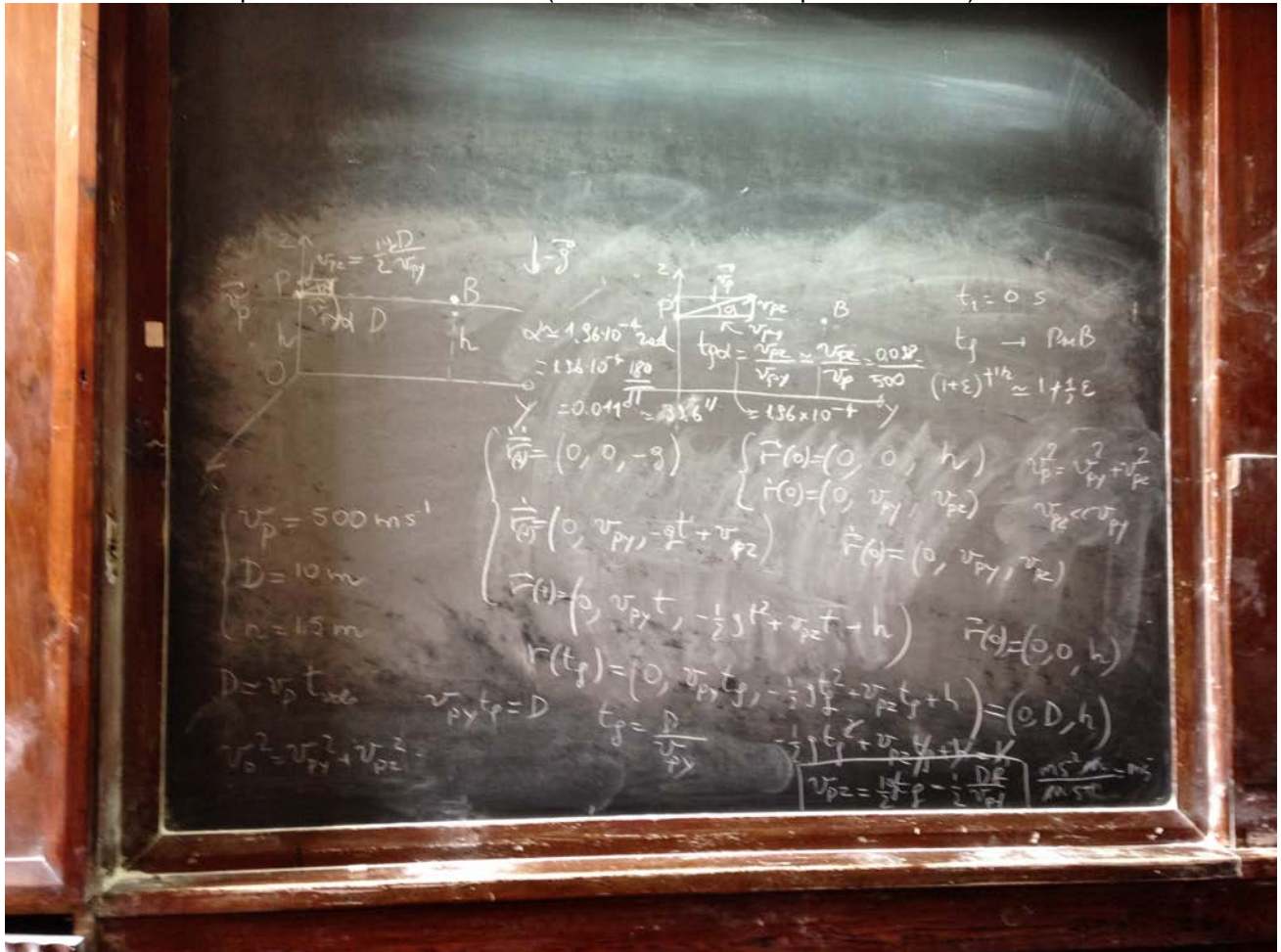
$x(t) = a \cos(\omega t + \alpha) = \text{Re}\{A e^{i\omega t}\}$ $A = a e^{i\alpha}$
 $F = -k(x_2 - x_1)$
 $E_{TOT} = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} k(x_2 - x_1)^2$
 $P = m_1 \dot{x}_1 + m_2 \dot{x}_2 = \text{costante}$
 $X_{DIFF} = x_2 - x_1$
 $x = x' + vt$ $\dot{x} = x' + v$
 $E_{TOT} = \frac{1}{2} m_1 \frac{m_2^2}{(m_1 + m_2)^2} \dot{X}_{DIFF}^2 + \frac{1}{2} m_2 \frac{m_1^2}{(m_1 + m_2)^2} \dot{X}_{DIFF}^2 + \frac{1}{2} k X_{DIFF}^2$
 $= \frac{1}{2} \dot{X}_{DIFF}^2 \left(\frac{m_1 m_2}{m_1 + m_2} \right) + \frac{1}{2} k X_{DIFF}^2$ $\mu = \frac{m_1 m_2}{m_1 + m_2}$
 $E_{TOT} = \frac{1}{2} \mu \dot{X}_{DIFF}^2 + \frac{1}{2} k X_{DIFF}^2 = X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$
 $\begin{cases} X_{DIFF} = x_2 - x_1 \\ m_1 x_1 + m_2 x_2 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = -\frac{m_2}{m_1} x_2 \\ x_1 = -\frac{m_2}{m_1} (X_{DIFF} + x_1) \end{cases}$

$e^{i\omega t} = \cos \omega t + i \sin \omega t$
 $\frac{d}{dt} e^{i\omega t} = i\omega e^{i\omega t}$
 $\frac{d}{dt} e^{i(\omega t + \alpha)} = i\omega e^{i(\omega t + \alpha)}$
 $A = a e^{i\alpha} \in \mathbb{C}$
 $\Rightarrow a \cos(\omega t + \alpha) + i a \sin(\omega t + \alpha)$
 $z = x + iy$
 $x_2 = \frac{m_1}{m_1 + m_2} X_{DIFF}$
 $x_1 = -\frac{m_2}{m_1 + m_2} X_{DIFF}$
 (28)

27 Marzo 2014 (Secondo compito)

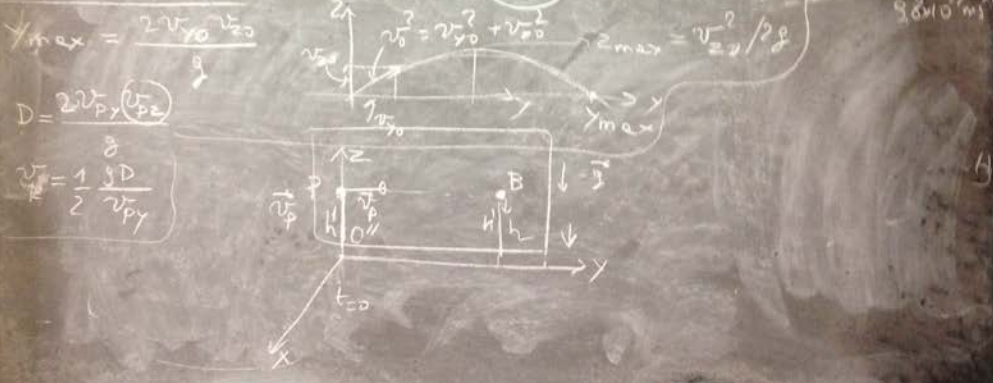
1 Aprile 2014

Correzione del compito del 27 Marzo 2014 (testo e soluzione disponibili in rete)



$$v_p^2 = v_{p1}^2 + v_{p2}^2 = v_p^2 \left(1 + \frac{v_p^2}{2g^2}\right) \quad v_p^2 = \frac{2g^2}{(1 + \frac{v_p^2}{2g^2})}$$

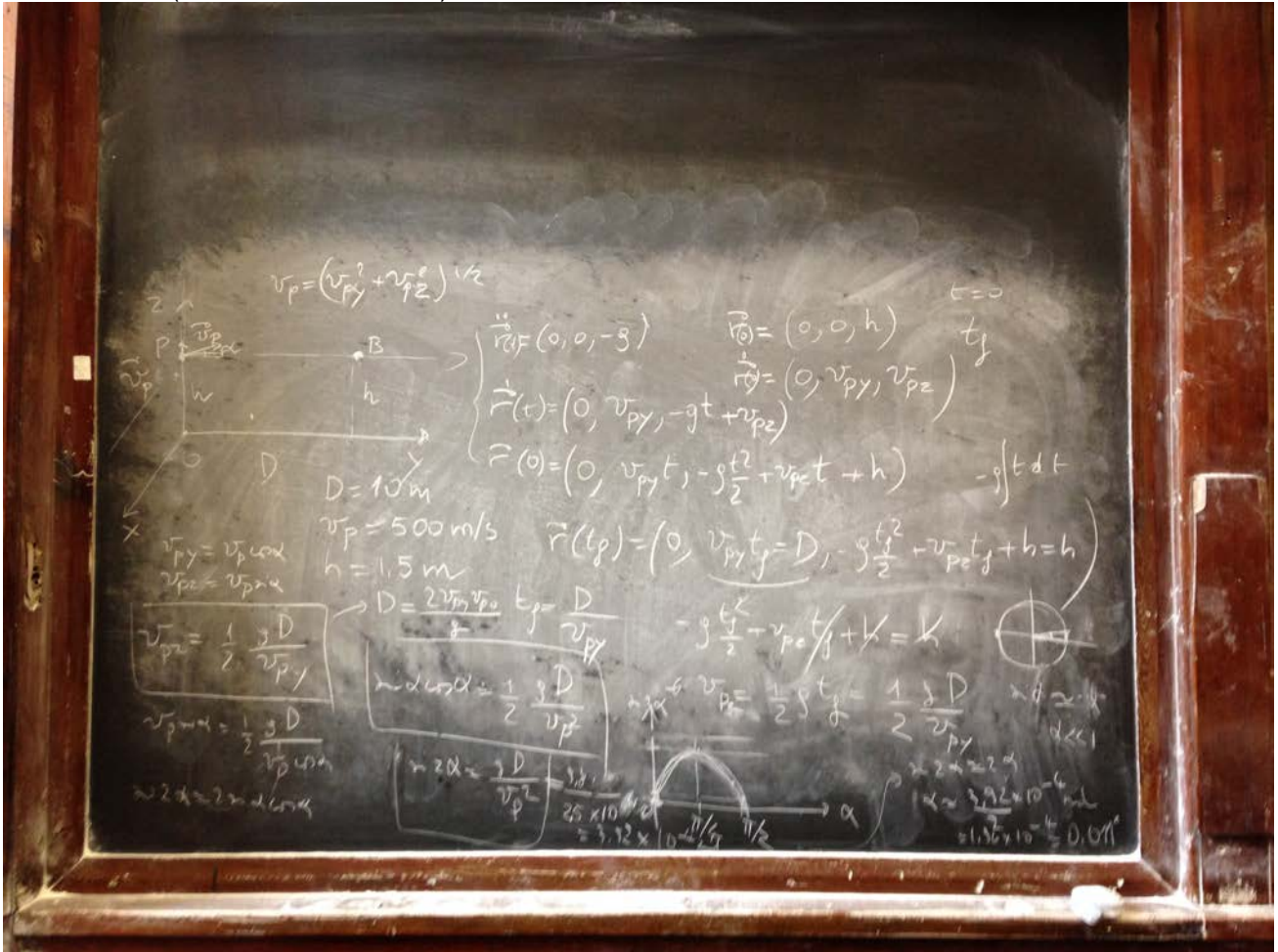
$$v_p = \frac{1}{2} \frac{gD}{v_p} \left(1 + \frac{v_p^2}{2g^2}\right) \approx \frac{1}{2} \frac{gD}{v_p} \left(1 + \frac{1}{2} \frac{v_p^2}{g^2}\right) \approx \frac{1}{2} \frac{gD}{v_p} \rightarrow \frac{1}{2} \frac{28 \cdot 10}{500} \approx 0.28 \text{ m/s}$$



$\vec{\omega} = \omega_{\oplus} \hat{z}$
 $\vec{r} = (2\omega_{\oplus} v_p r \vartheta, 0, 0)$
 $\vec{r}(0) = (0, 0, h)$
 $\dot{\vec{r}}(0) = (v_{px}, v_{py}, 0)$
 $\omega_{\oplus} = 7.29 \times 10^{-5} \text{ rad/s}$
 $2\omega_{\oplus} v_p \approx 2 \cdot 7.29 \times 10^{-5} \cdot 5 \times 10^2 \text{ ms}^{-2} = 7.29 \times 10^{-5} \cdot 10^3 \text{ ms}^{-2} \approx 7.29 \cdot 10^{-2} \text{ ms}^{-2}$
 $\vec{r}(t) = (2\omega_{\oplus} v_p r \vartheta t + v_{px} t, v_{py} t, 0)$
 $\vec{r}(t) = (\omega_{\oplus} v_p r \vartheta t^2 + v_{px} t, v_{py} t, h)$
 $\vec{r}(t_g) = (\omega_{\oplus} v_p r \vartheta t_g^2 + v_{px} t_g, v_{py} t_g, h)$
 $t_g = \frac{D}{v_{py}}$
 $-\omega_{\oplus} v_p r \vartheta \frac{D}{v_{py}} = -v_{px}$
 $v_{px} = -\omega_{\oplus} r \vartheta \frac{D}{v_{py}} \approx -\omega_{\oplus} r \vartheta D = -7.3 \times 10^{-5} \cdot 10^3 \cdot 10^3 \text{ ms}^{-1} \approx -7.3 \times 10^{-4} \text{ ms}^{-1}$

3 Aprile 2014

Continua soluzione del compitino del 27 marzo. Continua problema delle 2 masse accoppiate da una forza elastica (caso unidimensionale)



$$a_{cent} \approx 2\omega \cdot v_{KR} \approx 2 \cdot 7.3 \times 10^{-5} \cdot 5 \times 10^2 \text{ ms}^{-2} \approx 73 \times 10^{-2} \text{ ms}^{-2} = 0.73 \text{ ms}^{-2}$$

$$a_{cent} \approx \omega_0^2 R_0 = (0.31 \cdot 10^6)^2 \cdot 6.4 \times 10^6 \text{ ms}^{-2} = 34 \times 10^2 \text{ ms}^{-2}$$

$$\frac{a_{cent}}{\omega} = 3.5 \times 10^3 = 0.034 \text{ ms}^{-2}$$



Diagram of two masses m_1 and m_2 connected by a spring with spring constant k . The displacement of the center of mass is X . The relative displacement is $X_{DIFF} = x_2 - x_1$.

Equations of motion:

$$F = -kX$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$X_{DIFF} = x_2 - x_1$$

$$F = -k(x_2 - x_1)$$

$$m_1 \ddot{x}_1 = k(x_2 - x_1) = k X_{DIFF}$$

$$m_2 \ddot{x}_2 = -k(x_2 - x_1) = -k X_{DIFF}$$

$$X_1 = \frac{m_2}{m_1 + m_2} X_{DIFF}$$

$$X_2 = \frac{m_1}{m_1 + m_2} X_{DIFF}$$

$$m_1 m_2 (\ddot{x}_2 - \ddot{x}_1) = -k m_1 X_{DIFF} - k m_2 X_{DIFF}$$

$$\ddot{X}_{DIFF}$$

$$m_1 m_2 \ddot{X}_{DIFF} = -k X_{DIFF} (m_1 + m_2)$$

$$\frac{m_1 m_2}{m_1 + m_2} \ddot{X}_{DIFF} = -k X_{DIFF}$$

$$\omega \ddot{X}_{DIFF} = -k X_{DIFF}$$

$$\omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{2k}{m}}$$

$m_1 = m_2$

$$\ddot{X} = -\frac{k}{m} X_{DIFF}$$

$$E = \frac{1}{2} m \dot{X}_{DIFF}^2 + \frac{1}{2} k X_{DIFF}^2$$

$$E = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} k X_{DIFF}^2$$

$$E = \frac{1}{2} M \dot{X}_{DIFF}^2 + \frac{1}{2} k X_{DIFF}^2$$



(27)

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$m_1 = m_2 = m$$

$$\mu = \frac{m^2}{2m} = \frac{m}{2}$$

$$m_2 \ll m_1$$

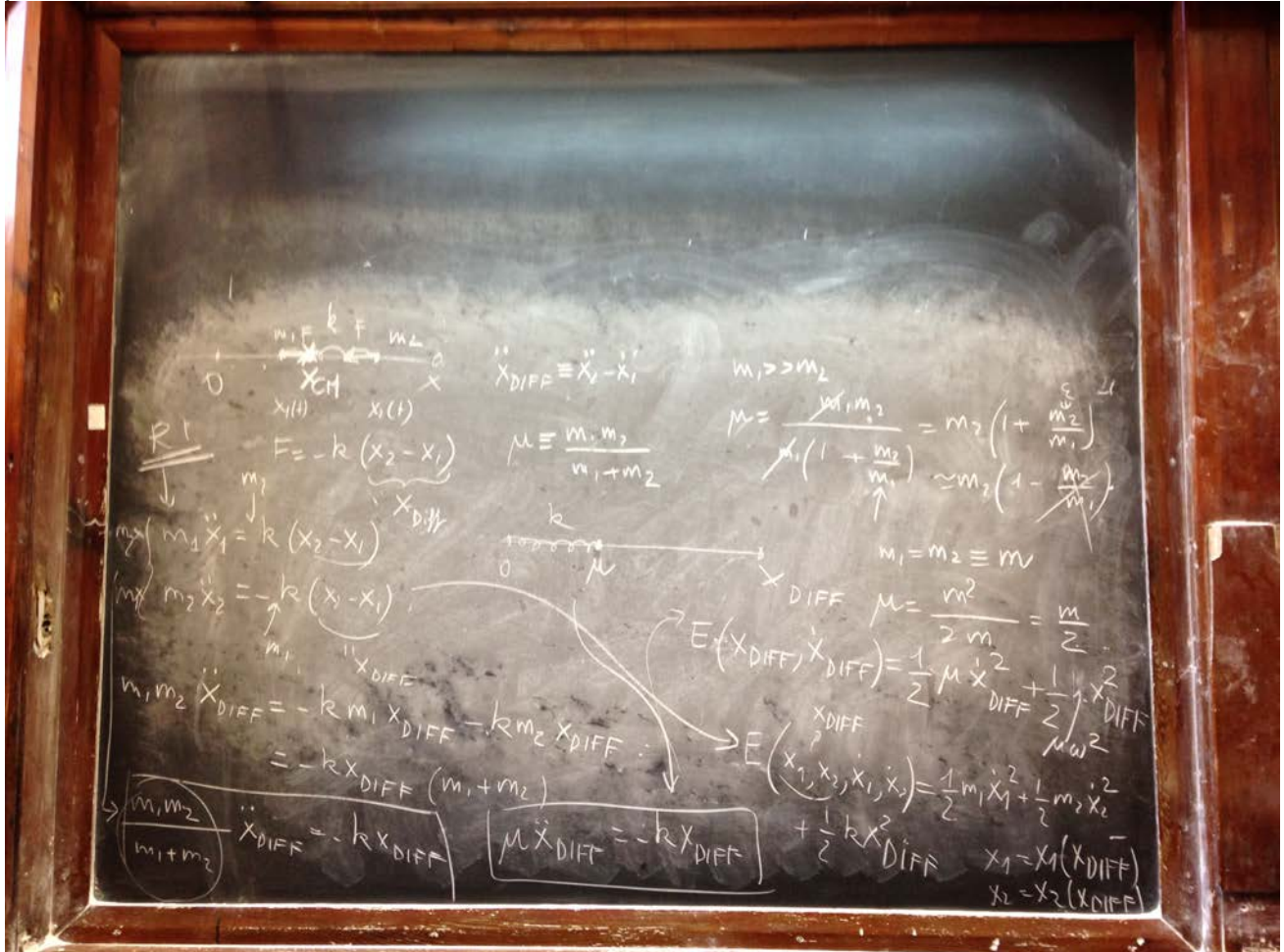
$$M = \frac{m_1 m_2}{m_1 + m_2} \approx m_2$$

10 Aprile 2014

Derivazione della massa ridotta in modo analogo nel caso di 2 masse accoppiate da una forza elastica e nel caso del problema dei 2 copri gravitazionale.

Problema della molecola biatomica formata da isotopi degli stessi atomi.

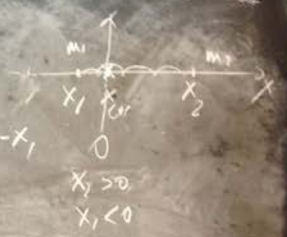
Concetto di dinamometro e calcolo della costante elastica equivalente per molle in serie e molle in parallelo.



CENTRO DI MASSA

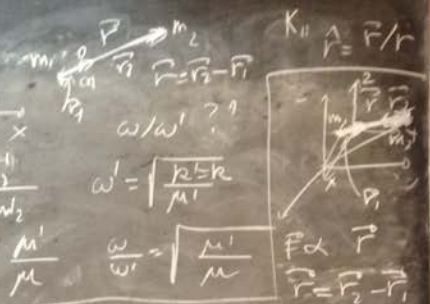
combinazione di coordinate

$$\begin{aligned}
 X_{CH} &\equiv \frac{m_1 X_1 + m_2 X_2}{m_1 + m_2} = 0 & \ddot{X}_{CH} &= 0 & \dot{X}_{CH} &= \text{cost} \\
 X_{DIFF} &\equiv X_2 - X_1 & & & & \\
 m_1 X_1 &= -m_2 X_2 & \frac{X_1}{X_2} &= -\frac{m_2}{m_1} & & \\
 X_{DIFF} &= X_2 - X_1 & X_{DIFF} &= X_2 + X_2 \frac{m_2}{m_1} = X_2 \left(1 + \frac{m_2}{m_1}\right) = X_2 \frac{m_1 + m_2}{m_1} \\
 \ddot{X}_2 &= \frac{m_1}{m_1 + m_2} \ddot{X}_{DIFF} & E(X_{DIFF}, \ddot{X}_{DIFF}) &= \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} \left(\frac{m_2 + m_1}{m_1 + m_2}\right)^2 X_{DIFF}^2 + \frac{1}{2} k X_{DIFF}^2 \\
 \ddot{X}_1 &= -\frac{m_2}{m_1 + m_2} \ddot{X}_{DIFF} & &= \frac{1}{2} \mu \ddot{X}_{DIFF}^2 + \frac{1}{2} k X_{DIFF}^2 \\
 E(X_{DIFF}, \ddot{X}_{DIFF}) &= \frac{1}{2} \frac{m_1 m_2}{(m_1 + m_2)^2} X_{DIFF}^2 + \frac{1}{2} \frac{m_2}{(m_1 + m_2)} \mu \ddot{X}_{DIFF}^2 + \frac{1}{2} k X_{DIFF}^2
 \end{aligned}$$

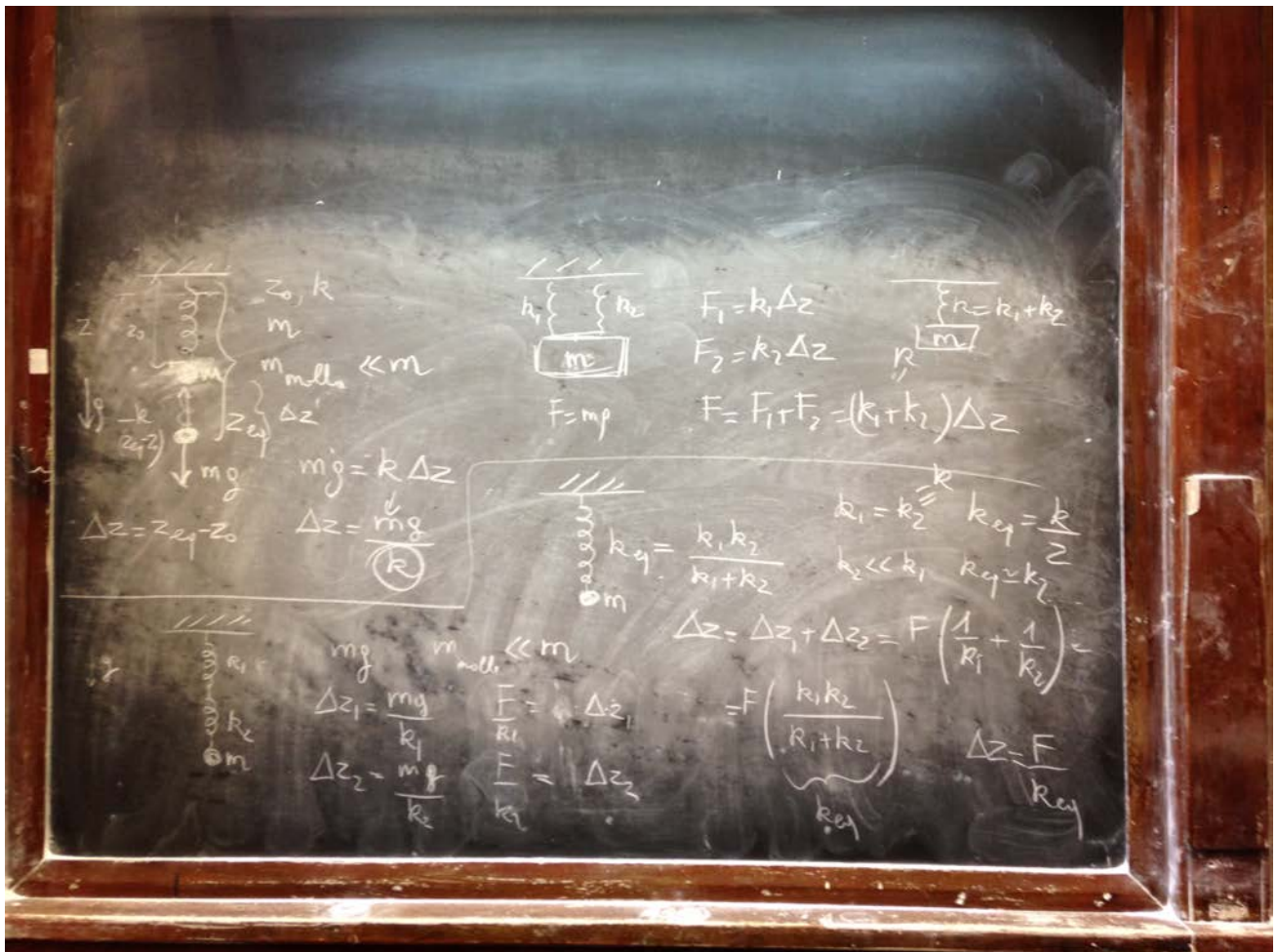


ORIGINE NEL CM

$$\begin{aligned}
 \mu \ddot{x} &= -kx & \omega &= \sqrt{\frac{k}{\mu}} \\
 \mu &= \frac{m_1 m_2}{m_1 + m_2} & \omega &= \sqrt{\frac{k(m_1 + m_2)}{m_1 m_2}} \\
 \omega^2 &= \frac{k}{\mu} = \frac{k}{m_1 m_2} (m_1 + m_2) & & \\
 \vec{R}_{CH} &= 0 & \vec{r}_1 &= -\frac{m_2}{m_1} \vec{r}_2 & \frac{|\vec{r}_1|}{|\vec{r}_2|} &= \frac{m_2}{m_1} \\
 \vec{r}_1 &= \frac{m_2}{m_1 + m_2} \vec{r} & \vec{r}_2 &= \frac{m_1}{m_1 + m_2} \vec{r} & & \\
 \vec{F}_1 &= -\frac{m_2}{m_1 + m_2} \frac{GM(m_1 + m_2)}{r^3} \vec{r} & \vec{F}_2 &= -\frac{m_1}{m_1 + m_2} \frac{GM(m_1 + m_2)}{r^3} \vec{r} & & \\
 \mu \ddot{\vec{r}} &= -\frac{GM(m_1 + m_2)}{r^3} \vec{r} & & & & \\
 \mu &= \frac{m_1 m_2}{m_1 + m_2} & & & & \\
 \ddot{\vec{r}} &= -\frac{GM(m_1 + m_2)}{r^3} \vec{r} & & & &
 \end{aligned}$$



20



6 Maggio 2014

Oscillatore armonico forzato: soluzione mediante l'uso di esponenziali complessi con particolare attenzione alla rappresentazione grafica dei numeri complessi.

Risposta dell'oscillatore, in ampiezza e fase, in funzione della frequenza forzante

Concetto di risonanza (caso ideale)

$m, k \xrightarrow{\frac{k}{0 \times m}} x$ $\ddot{x} + \omega_0^2 x = 0$ $\omega_0 = \sqrt{k/m}$ $x(t) = A \cos(\omega_0 t + \phi)$
 $m\ddot{x} = -kx$ $E(x, \dot{x}) = \frac{1}{2} k x^2 + \frac{1}{2} m \dot{x}^2$ $\dot{x} = -A \omega_0 \sin(\omega_0 t + \phi)$
 Oscillateur libre $E = \frac{1}{2} m \omega_0^2 A^2 (\cos^2(\omega_0 t + \phi) + \sin^2(\omega_0 t + \phi)) = \frac{1}{2} m \omega_0^2 A^2$ $\beta = \sqrt{2 \times E}$
 $\rightarrow m\ddot{x} = -kx + F(t)$ $F(t) = F_0 \cos(\omega t + \Delta)$ $\rho = \frac{F_0}{m} = \frac{F_0}{k} \omega^2$ $z = x + iy$
 $\ddot{x} + \omega_0^2 x = \frac{F}{m} = \frac{F_0}{m} \cos(\omega t + \Delta)$ $\rho = -1$ $\tilde{z} = \dot{z} = \dot{x} + i\dot{y}$
 $\ddot{z} + \omega_0^2 z = \frac{f(t)}{m} = \frac{F_0}{m} e^{i(\omega t + \Delta)}$ $\tilde{z} = \dot{z} = \dot{x} + i\dot{y}$
 $\rightarrow z(t) = \frac{1}{\omega} e^{i(\omega t + \Delta)} = \frac{1}{\omega} (\cos(\omega t + \Delta) + i \sin(\omega t + \Delta))$ $\tilde{z} = \dot{z} = \dot{x} + i\dot{y}$
 $\frac{d}{dt} z(t) = i \omega z(t) = i \omega \frac{1}{\omega} e^{i(\omega t + \Delta)} = i e^{i(\omega t + \Delta)}$ $-1 = e^{i\pi} = e^{i(\omega t + \Delta) - \pi} = -1$

$-\omega^2 p e^{i\omega t} + \omega_0^2 p e^{i\omega t} = \frac{F_0}{m} e^{i\omega t} e^{i\Delta}$ $\rho = \frac{F_0/m}{\omega_0^2 - \omega^2} e^{i\Delta} = r e^{i\delta}$
 $\rho = \frac{F_0/m}{\omega_0^2 - \omega^2}$ $\delta = \Delta$ $\omega = \omega_0$
 $z(t) = \frac{F_0/m}{\omega_0^2 - \omega^2} e^{i(\omega t + \Delta)}$ $\omega > \omega_0$ $\omega^2 > \omega_0^2$
 $x(t) = \frac{F_0/m}{\omega_0^2 - \omega^2} \cos(\omega t + \Delta)$ $z(t) = \frac{F_0/m}{\omega_0^2 - \omega^2} e^{i(\omega t + \Delta)}$
 $\omega < \omega_0$ $\omega^2 < \omega_0^2$ $z(t) = \frac{F_0/m}{\omega_0^2 - \omega^2} e^{i(\omega t + \Delta)}$

Osc. forzato mazzato

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = F(t)$$

$$(\gamma \dot{x})' = m s^{-2} \cdot \gamma = \frac{m s^{-2}}{m s^{-1}} = s^{-1}$$

$$K_R = \frac{1}{\gamma}$$

$$\ddot{z} + \gamma \dot{z} + \omega_0^2 z = \frac{F_0}{m} e^{i(\omega t + \Delta)}$$

$$z(t) = p e^{i(\omega t + \delta)} = p e^{i\delta} e^{i\omega t}$$

$$\omega_0 \gamma = \frac{2\pi}{T}$$

$$- \omega^2 p e^{i\delta} e^{i\omega t} + i \omega \gamma p e^{i\delta} e^{i\omega t} + \omega_0^2 p e^{i\delta} e^{i\omega t} = \frac{F_0}{m} e^{i\Delta} e^{i\omega t}$$

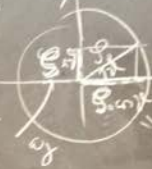
$$p e^{i\delta} (\omega_0^2 - \omega^2 + i \omega \gamma) = \frac{F_0}{m} e^{i\Delta}$$

$$x e^{-i\omega t} = \omega \Delta + i \pi \Delta$$

$$p e^{i\delta} (\omega_0^2 - \omega^2 + i \omega \gamma) = \frac{F_0}{m} e^{i\Delta}$$

$$p e^{i\delta} = \frac{F_0/m}{\omega_0^2 - \omega^2 + i \omega \gamma} e^{i\Delta} = \frac{F_0}{\omega_0} e^{i\delta}$$

$$(\omega_0^2 - \omega^2) + i \omega \gamma = \frac{F_0}{m} e^{i\delta}$$



$$= \frac{F_0/m}{\omega_0} e^{i\delta} = \frac{F_0/m}{\omega_0} e^{-i\delta} e^{i\Delta} = \frac{F_0/m}{\omega_0} e^{i(\Delta - \delta)}$$

$$\omega_0 = \sqrt{(\omega_0^2 - \omega^2)^2 + \omega^2 \gamma^2}$$

$$t_{gr} = \frac{\omega \delta}{\omega_0^2 - \omega^2}$$

$$= \frac{F_0/m}{\omega_0} e^{i(\Delta - \delta)}$$

$$t_g(\delta) = -t_{gr}$$

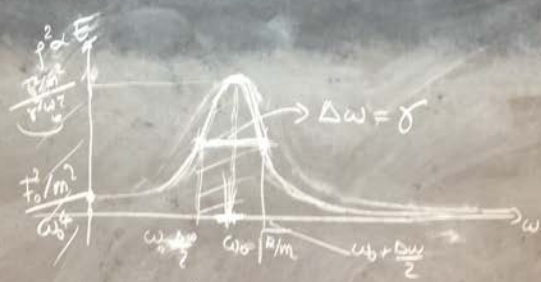
$$e^{i\delta} = \frac{F_0/m}{\omega_0} e^{i(\Delta - \delta)}$$

$$\boxed{p^2 = \frac{F_0^2/m^2}{(\omega_0^2 - \omega^2)^2 + \omega^2 \gamma^2}}$$

$$p^2 = \frac{F_0^2/m^2}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}$$

$$\omega = \omega_0 \quad p^2(\omega = \omega_0) = \frac{F_0^2/m^2}{\gamma^2 \omega_0^2}$$

$$p^2(\omega = 0) = \frac{F_0^2/m^2}{\omega_0^4}$$



Caso γ molto piccolo; $(\omega_0^2 - \omega^2)^2$ domina rispetto a $\gamma^2 \omega^2$

$$(\omega_0^2 - \omega^2) = (\omega_0 - \omega)(\omega_0 + \omega) \approx 2\omega_0(\omega_0 - \omega)$$

(22)

$$p^2 \approx \frac{F_0^2/m^2}{4\omega_0^3(\omega_0 - \omega)^2 + \gamma^2 \omega^2} = \frac{F_0^2}{4m^2 \omega_0^3 [(\omega_0 - \omega)^2 + \frac{\gamma^2}{4}]}$$

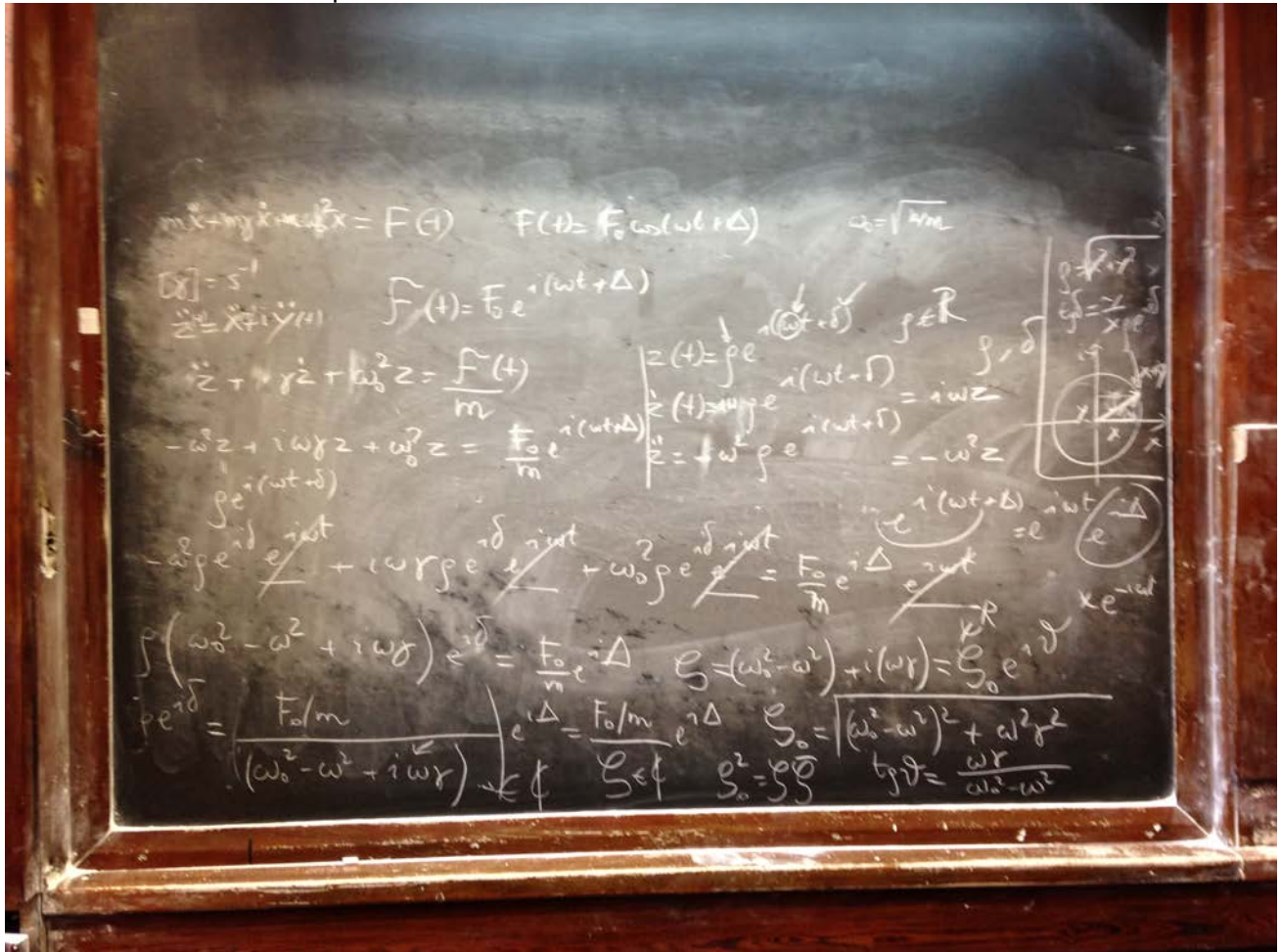
8 Maggio 2014

Oscillatore armonico forzato e smorzato risolto mediante passaggio agli esponenziali complessi.
 Concetto di dissipazione.

Risposta dell'oscillatore. Ruolo della risonanza. Approssimazione nel caso di piccola dissipazione.

Concetto e definizione del fattore di qualità Q e sua relazione con il coefficiente di dissipazione.

Ruolo del Q e sua misura sperimentale



$$F_0 e^{i\delta} = \frac{F_0/m}{\omega_0^2 - \omega^2} e^{i\delta} = \frac{F_0}{m\omega_0^2} e^{-i\delta} \omega = \frac{F_0}{m\omega_0^2} e^{i(\Delta - \delta)}$$

$$\beta = \frac{F_0}{m\omega_0^2} \quad E \propto A^2 = \beta^2 \quad \beta^2 = \frac{F_0^2/m^2}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2}$$

$$\gamma = \Delta + \delta$$

$$t_2 - t_1 = -t_2 \delta = -\frac{\omega \delta}{\omega_0^2 - \omega^2}$$

$$(\omega_0^2 - \omega^2) = (\omega_0 - \omega)(\omega_0 + \omega) = 2\omega_0(\omega_0 - \omega)$$

$$\beta^2 = \frac{F_0^2/m^2}{4\omega_0^2(\omega_0 - \omega)^2 + \gamma^2 \omega^2} = \frac{F_0^2}{4m^2 \omega_0^2 \left[(\omega_0 - \omega)^2 + \frac{\gamma^2 \omega^2}{4\omega_0^2} \right]}$$

$$\omega_2 - \omega_1 = \gamma$$

$$E(t) = E_0 e^{-\gamma t} \quad t=0 \quad E(0) = E_0 \quad t_1 = \frac{1}{\gamma} \quad E\left(\frac{1}{\gamma}\right) = E_0 e^{-1} = \frac{E_0}{e}$$

$$= E_0 e^{-t/\tau} \quad \tau = 1/\gamma \quad t_2 = 2/\gamma \quad E\left(\frac{2}{\gamma}\right) = E_0 e^{-2} = \frac{E_0}{e^2}$$

$$E(t) = E_0 e^{-\omega_0 t/Q} \quad \gamma = \frac{\omega_0}{Q} \quad t_n = n \frac{1}{\gamma} \quad E\left(\frac{n}{\gamma}\right) = E_0 e^{-n} = \frac{E_0}{e^n}$$

$$\frac{dE}{dt} = -\frac{\omega_0 E_0}{Q} e^{-\omega_0 t/Q} = -\frac{\omega_0}{Q} E$$

$$\frac{dE}{E} = -\frac{\omega_0 dt}{Q} \quad \Delta t = P_0$$

$$\frac{\Delta E_{P_0}}{E} = -\frac{\omega_0 P_0}{Q} = -\frac{2\pi}{Q}$$

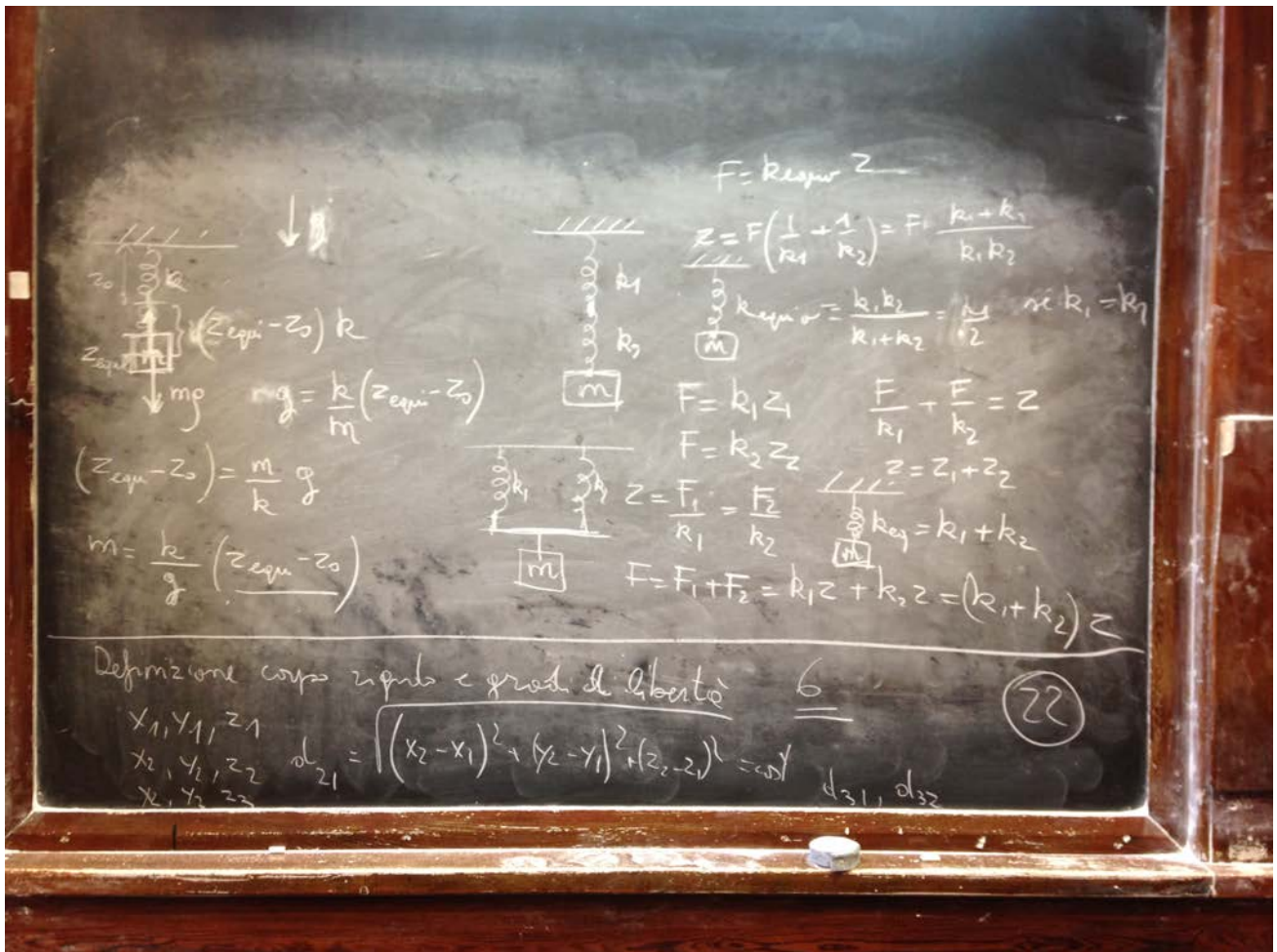
$$Q = \frac{E_{P_0}}{(-\Delta E)_{P_0}} \quad \frac{1}{Q} = \frac{1}{2\pi} \frac{(-\Delta E)_{P_0}}{E_{P_0}}$$

$$E \propto A^2 \quad A \propto E^{1/2} \quad A(t) = A_0 \left(e^{-\omega_0 t/Q} \right)^{1/2} = A_0 e^{-\omega_0 t/2Q}$$

$$A_1 = A_0 e^{-\omega_0 t_1/2Q} \quad A_2 = A_0 e^{-\omega_0 t_2/2Q} \quad A_1 > A_2$$

$$\frac{A_1}{A_2} = \frac{A_0 e^{-\omega_0 t_1/2Q}}{A_0 e^{-\omega_0 t_2/2Q}} = e^{\frac{\omega_0}{2Q}(t_2 - t_1)}$$

$$\ln \frac{A_1}{A_2} = \frac{\omega_0}{2Q}(t_2 - t_1) \Rightarrow Q = \frac{\omega_0(t_2 - t_1)}{2 \ln(A_1/A_2)}$$



13 Maggio 2014

Oscillatore armonico semplicemente smorzato.

Corpo rigido. Gradi di libertà. Centro di massa, angoli di Eulero.

(soluzioni)
 Oscillatore armonico

$$m\ddot{x} + r\dot{x} + kx = 0$$

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0$$

$$\ddot{z} + \gamma\dot{z} + \omega_0^2 z = 0$$

$$(-\lambda^2 - i\gamma\lambda + \omega_0^2)z_0 e^{\lambda t} = 0$$

$$-\lambda^2 + i\gamma\lambda + \omega_0^2 = 0$$

$$\frac{k}{m} = \omega_0^2 \quad [\gamma] = s^{-1}$$

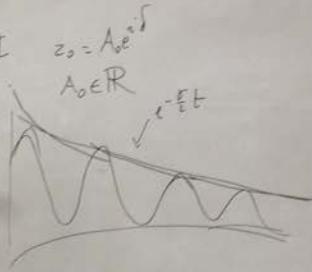
$$z = z_0 e^{i\omega t} \quad \alpha \in \mathbb{C}$$

$$\dot{z} = i\omega z_0 e^{i\omega t}$$

$$\ddot{z} = -\omega^2 z_0 e^{i\omega t}$$

$$z_0 = 0 \text{ non interessa}$$

$$e^{-\frac{\gamma}{2}t}$$



$$\alpha_{1,2} = \frac{i\gamma}{2} \pm \sqrt{\left(\frac{i\gamma}{2}\right)^2 + \omega_0^2} = i\frac{\gamma}{2} \pm \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}$$

$$\frac{\gamma^2}{4} < \omega_0^2$$

$$\sqrt{\omega_0^2 - \frac{\gamma^2}{4}} = \omega_0 \sqrt{1 - \frac{\gamma^2}{4\omega_0^2}} = \omega_0 \left(1 - \frac{\gamma^2}{8\omega_0^2}\right) = \omega_d$$

$$\alpha_1 = i\frac{\gamma}{2} + \omega_d$$

$$\alpha_2 = i\frac{\gamma}{2} - \omega_d$$

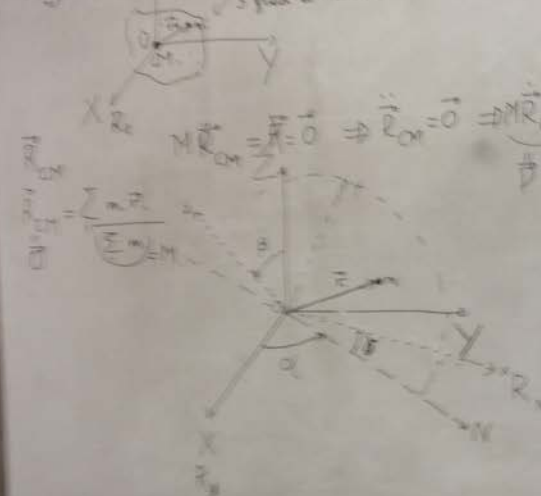
$$z_1 = z_0 e^{(i\frac{\gamma}{2} + \omega_d)t} = z_0 e^{-\frac{\gamma}{2}t} e^{i\omega_d t}$$

$$z_2 = z_0 e^{(i\frac{\gamma}{2} - \omega_d)t} = z_0 e^{-\frac{\gamma}{2}t} e^{-i\omega_d t}$$

$$z = z_1 + z_2 = z_0 e^{-\frac{\gamma}{2}t} (e^{i\omega_d t} + e^{-i\omega_d t}) = A_0 e^{-\frac{\gamma}{2}t} (e^{i(\omega_d t + \delta)} + e^{-i(\omega_d t + \delta)}) = 2A_0 e^{-\frac{\gamma}{2}t} \cos(\omega_d t + \delta) \in \mathbb{R}$$

$$e^{-t/\tau} \quad \tau = \frac{2}{\gamma}$$

o gradi di libertà
 3 gradi di libertà



Rot.:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix}$$

$$R_{\alpha} = \begin{pmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_{\beta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & \sin \beta \\ 0 & -\sin \beta & \cos \beta \end{pmatrix}$$

$$R_{\gamma} = \begin{pmatrix} \cos \gamma & \sin \gamma & 0 \\ -\sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = R_{\gamma} R_{\beta} R_{\alpha} \begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix}$$

$$R_{\alpha} R_{\beta} R_{\gamma} = \begin{pmatrix} R_{\alpha} \\ R_{\beta} \\ R_{\gamma} \end{pmatrix}$$

$$T = \frac{1}{2} \sum_{i=1}^N m_i \dot{\vec{r}}_i \cdot \dot{\vec{r}}_i = \frac{1}{2} \sum_{i=1}^N m_i \dot{\vec{r}}_i \cdot (\omega \wedge \vec{r}_i) = \frac{1}{2} \sum_{i=1}^N m_i \omega \cdot (\vec{r}_i \times \dot{\vec{r}}_i)$$

$$= \frac{1}{2} \omega \cdot \vec{L}$$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{c} \cdot (\vec{a} \times \vec{b}) = \vec{b} \cdot (\vec{c} \times \vec{a})$$

$$(\vec{b} \times \vec{c}) \cdot \vec{a}$$

$$\vec{\omega} = \omega \hat{n}$$

quantità di moto
 angolare della massa
 m_i

$$\vec{L} = \sum m_i \vec{r}_i \times \dot{\vec{r}}_i = \sum m_i \vec{r}_i \times (\omega \wedge \vec{r}_i)$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$$

$$\left(\frac{d}{dt}\right)_{R_1} = \left(\frac{d}{dt}\right)_{R_{NI}} + \vec{\omega} \times \vec{r}$$

$$\vec{r}_i = \vec{\omega} \times \vec{r}_i$$

$$\left(\frac{d\vec{\omega}}{dt}\right)_{R_1} = \left(\frac{d\vec{\omega}}{dt}\right)_{R_{NI}}$$

$$T = \frac{1}{2} \omega^2 \left(\sum m_i (r_i^2 - (\hat{n} \cdot \vec{r}_i)^2) \right) = \frac{1}{2} I_A \omega^2 = T$$

$$T = \frac{1}{2} m v^2$$

$$I_{\hat{n}} = \sum_i m_i (r_i^2 - (\hat{n} \cdot \vec{r}_i)^2)$$

$$\vec{L} = \sum_i m_i (\vec{r}_i \times \dot{\vec{r}}_i) = \sum_i m_i (\vec{r}_i \times \omega \wedge \vec{r}_i)$$

$$L_x = \sum_i m_i r_i^2 \omega_x - \omega_x \sum_i m_i x_i^2 - \omega_y \sum_i m_i x_i y_i - \omega_z \sum_i m_i x_i z_i$$

$$L_y = \sum_i m_i r_i^2 \omega_y - \omega_y \sum_i m_i y_i^2 - \omega_x \sum_i m_i x_i y_i - \omega_z \sum_i m_i y_i z_i$$

$$L_z = \sum_i m_i r_i^2 \omega_z - \omega_z \sum_i m_i z_i^2 - \omega_x \sum_i m_i x_i z_i - \omega_y \sum_i m_i y_i z_i$$

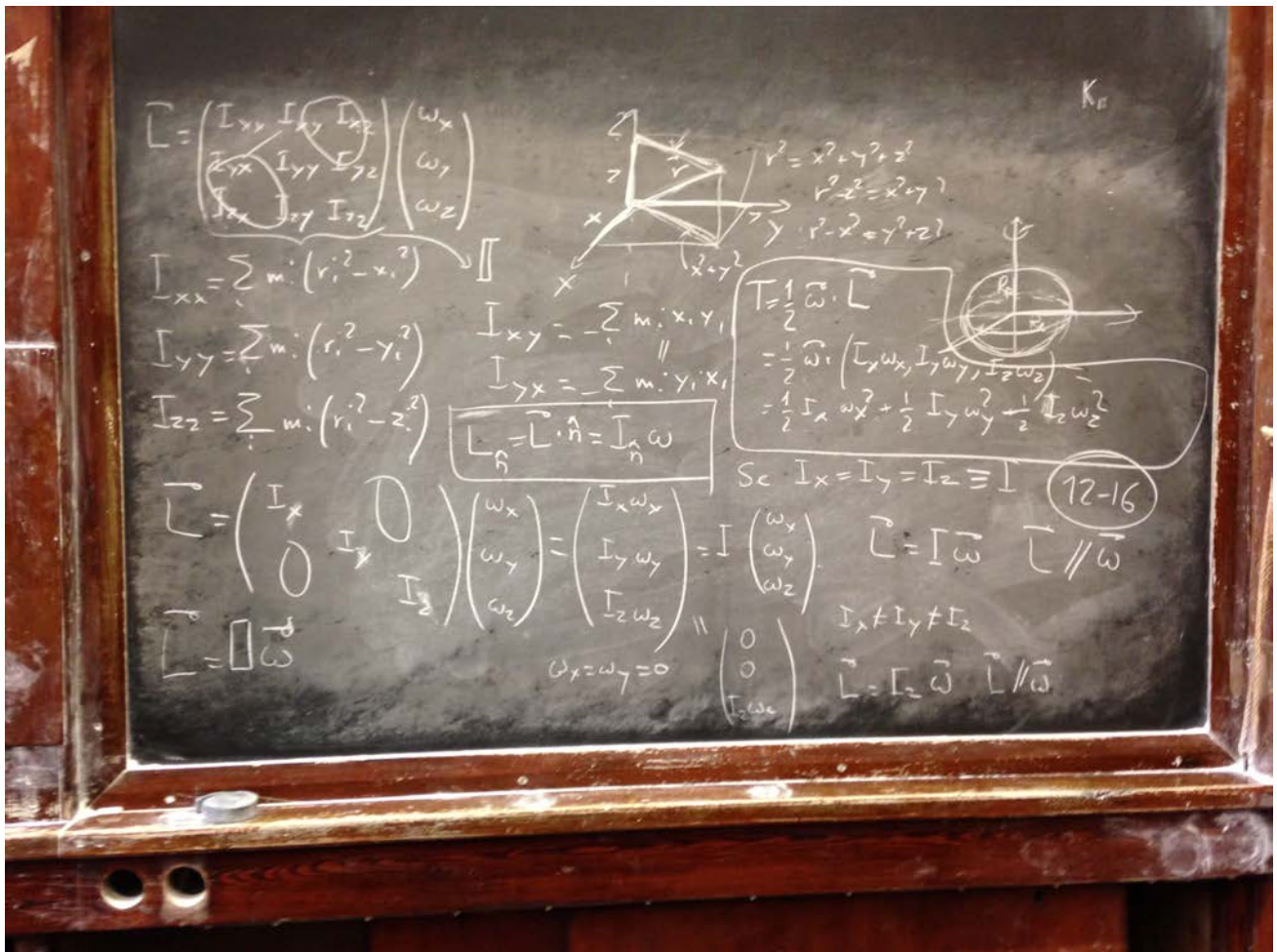
$$I_{\hat{n}} \equiv m_i d^2 = m_i (r_i^2 - (\vec{r}_i \cdot \hat{n})^2)$$

$$r_i^2 = x_i^2 + y_i^2 + z_i^2$$

$$r_i^2 - x_i^2 = y_i^2 + z_i^2$$

$$\vec{r}_i \times \hat{n} = |\vec{r}_i \times \hat{n}| = \sqrt{r_i^2 - (\vec{r}_i \cdot \hat{n})^2}$$

CM $\vec{r}_i = r_i \hat{n}$



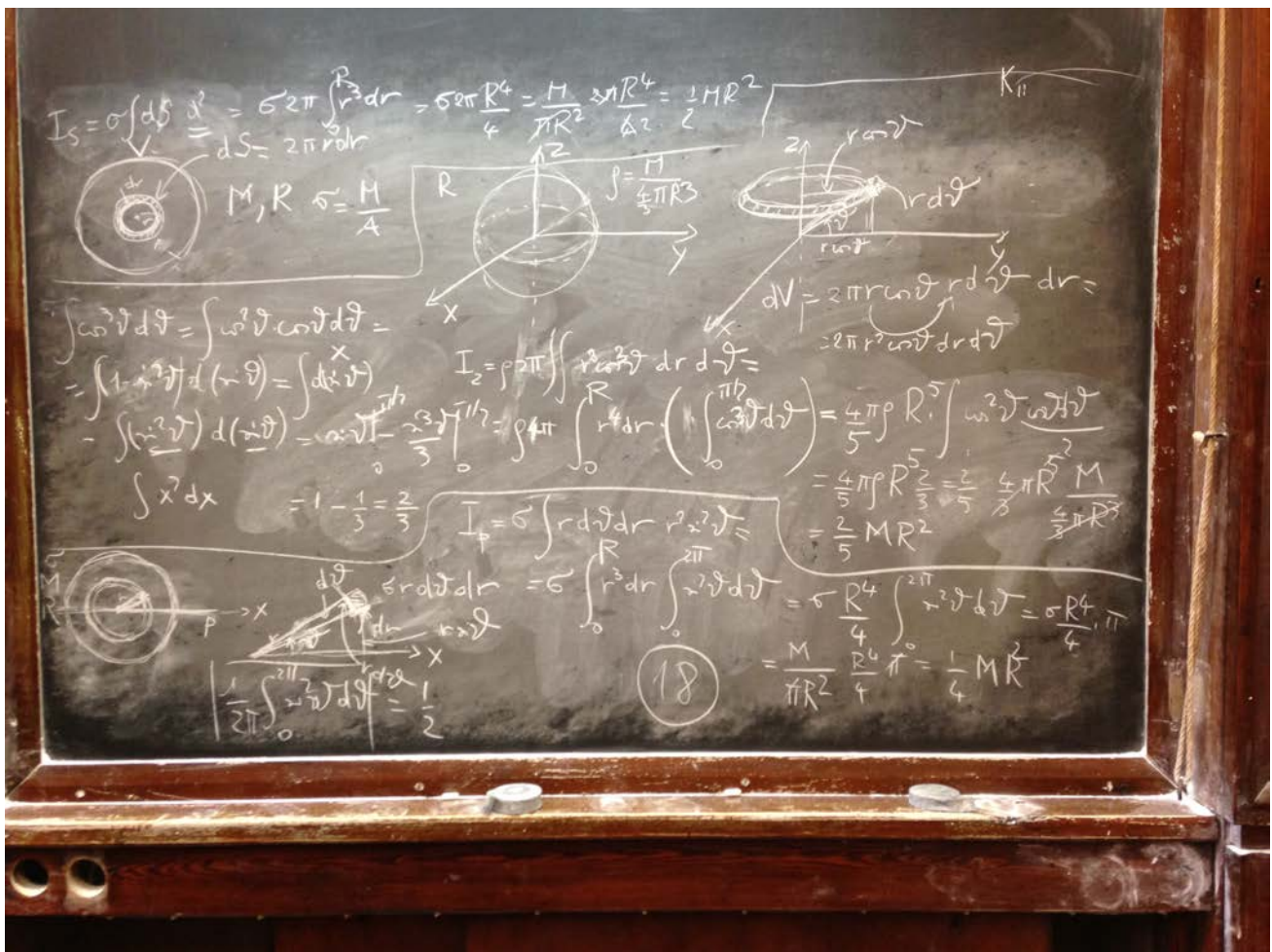
15 Maggio 2014

Corpo rigido. Matrice di inerzia. Assi principali di inerzia.

Momento d'inerzia. Metodi per il calcolo dei momenti di inerzia in vari casi significativi.

$\sum m_i \vec{r}_i \times \dot{\vec{r}}_i = \vec{L}$
 $(\dot{\vec{r}}_i) = \vec{\omega} \times \vec{r}_i$
 $(\dot{\vec{r}}_i)_{R_I} = 0$
 $(\dot{\vec{r}}_i)_{R_{CM}} = 0$
 $\vec{L} = \sum m_i \vec{r}_i \times \dot{\vec{r}}_i = \sum m_i \vec{r}_i \times (\vec{\omega} \times \vec{r}_i)$
 $I_a = \sum m_i d_i^2 = \sum m_i |\vec{r}_i \times \hat{n}_a|^2$
 $I = \sum_{sp} \sum m_i |\vec{r}_i' \times \hat{n}_a|^2$
 $\vec{F}_{CM} = \frac{\sum m_i \vec{r}_i'}{\sum m_i}$
 $\vec{0} = \frac{\sum m_i \vec{r}_i'}{\sum m_i}$
 $\vec{L}_{CM} = \vec{L} \cdot \hat{n} = I_a(\omega)$

$I_p = \sum m_i ((\vec{r}_i + \vec{r}_{CM}) \times \hat{n}_a)^2 = \sum m_i (\vec{r}_i \times \hat{n}_a)^2 + (\vec{r}_{CM} \times \hat{n}_a)^2 M + 2(\vec{r}_{CM} \times \hat{n}_a) \sum m_i (\vec{r}_i \times \hat{n}_a)$
 $= I_{CM} + M(\vec{r}_{CM} \times \hat{n}_a)^2 + 2(\vec{r}_{CM} \times \hat{n}_a) (\sum m_i \vec{r}_i \times \hat{n}_a)$
 $T = \frac{1}{2} \vec{\omega} \cdot \vec{L} = \frac{1}{2} I_a \omega^2$
 $\rho = \frac{M}{V}$
 $V = \pi R^2 h$
 $M = \rho V = \rho \pi R^2 h$
 $\rho = \frac{M}{\pi R^2 h}$
 $I_s = \int \rho r^2 dV = \rho \int_0^R \int_0^{2\pi} \int_0^h r^2 dr d\phi dz = \rho \pi h \int_0^R r^3 dr = \rho \pi h \frac{R^4}{4}$
 $I_s = \frac{M}{\pi R^2 h} \pi h \frac{R^4}{4} = \frac{1}{4} M R^2$
 $I_s = \frac{1}{2} M R^2$



20 Maggio 2014 (3 ore)

Equazioni di Newton del moto del corpo rigido

Riferimento inerziale e riferimento solidale con il corpo rigido. Derivate rispetto al tempo nei due sistemi.

Equazioni di Eulero del moto del corpo rigido attorno al proprio centro di massa.

Loro applicazione nel caso della precessione libera della Terra come corpo rigido

$\vec{L} = \vec{r} \times \vec{p}$ $(\vec{p} = \vec{F})$ $\vec{M}_0 = \vec{r} \times (m\vec{g}) = m \vec{r} \times \vec{g}$ \perp plano, vertical
 $\vec{M} = m g r \sin \theta$ $\vec{L} = \vec{r} \times (m\vec{v})$ \perp plano, uscente
 $m l \dot{\theta} = L = (m l v)$ " $l \hat{e}_\rho \times (m l \dot{\theta} \hat{e}_\theta) =$
 $\vec{L} = l \hat{e}_\rho \times (m l \dot{\theta} \hat{e}_\theta) =$
 $= m l^2 \dot{\theta} \hat{e}_\rho \times \hat{e}_\theta = m l^2 \dot{\theta} \hat{e}_\phi$
 $m l^2 \ddot{\theta} = -m g r \sin \theta$
 $m = \rho V = \rho \pi R^2 h$ $\vec{\omega} = \omega \hat{n}$
 $L = I \omega$
 $\vec{N} = -m g \sin \theta$

$\dot{\theta} = -\frac{g}{l} \sin \theta$
 $\ddot{\theta} = -\frac{g}{l} \cos \theta$
 $\ddot{\theta} + \frac{g}{l} \sin \theta = 0$ $\omega_0 = \frac{g}{l}$ $P_0 = 2\pi \sqrt{\frac{l}{g}}$ $l = 1m$

$\vec{L}_0 = I_0 \dot{\omega}$
 $I_0 \ddot{\omega} = -m g \sin \theta \frac{h}{2}$
 $\ddot{\omega} = -\frac{m g h \sin \theta}{I_0}$
 $\ddot{\omega} + \omega^2 = 0$ $\omega^2 = \frac{m g h \sin \theta}{I_0}$

$I_0 = I_{cm} + m \frac{h^2}{4} = \frac{1}{2} m R^2 + \frac{1}{4} m h^2 = \frac{1}{4} m R^2 + \frac{1}{3} m h^2$
 $I_{cm} = \frac{1}{2} m (R^2 + h^2)$

$\omega^2 = \frac{2}{\frac{2R^2}{3} + \frac{1}{3} h^2}$

$$\dot{\vec{L}} = \begin{pmatrix} I_x \dot{\omega}_x \\ I_y \dot{\omega}_y \\ I_z \dot{\omega}_z \end{pmatrix} + \vec{\omega} \times \vec{L} = \vec{N}$$

$$\left(\frac{d}{dt} \right)_{R_I} = \left(\frac{d}{dt} \right)_{R_{NI}} + \vec{\omega} \times$$

$$\begin{cases} I_x \dot{\omega}_x - \omega_y \omega_z (I_y - I_z) = N_x \\ I_y \dot{\omega}_y - \omega_z \omega_x (I_z - I_x) = N_y \\ I_z \dot{\omega}_z - \omega_x \omega_y (I_x - I_y) = N_z \end{cases}$$

Eq. Euler's $(\vec{L} = \vec{N})_{R_I}$
 x, y, z are principal axes of inertia of the rigid body

$I_x = I_y = I_x$ $N_x = N_y = N_z = 0$
 $I_z > I_x$

$$\begin{cases} I_x \dot{\omega}_x - \omega_y \omega_z (I_y - I_z) = 0 \\ I_y \dot{\omega}_y - \omega_z \omega_x (I_z - I_x) = 0 \\ I_z \dot{\omega}_z = 0 \end{cases}$$

$\dot{\omega}_z = 0 \Rightarrow \omega_z = \text{costante}$
 $\Omega = \omega_z \left(\frac{I_x - I_z}{I_x} \right)$

$$\begin{cases} \dot{\omega}_x = \omega_y \Omega \\ \dot{\omega}_y = -\omega_x \Omega \end{cases}$$

$$\begin{cases} \dot{\omega}_x = \omega_y \Omega - \Omega \omega_x \\ \dot{\omega}_y = -\omega_x \Omega - \Omega \omega_y \end{cases}$$

$$\dot{\omega}_x + \Omega^2 \omega_x = 0$$

$\omega_z = \text{costante}$
 $\Omega = \omega_z \frac{I_x - I_z}{I_x}$
 $|\Omega| < \omega_z$
 $I_z > I_x$ Ω retro precesso a quello di ω_z

$T = \left(\frac{2\pi}{\omega_z} \right) \frac{I_x}{I_z - I_x} > 1 \text{ gms}$ $T \approx 300 \text{ gms}$
 $\frac{I_x}{I_z - I_x} \approx 300$ \downarrow approssimo corpo rigido
 $m = 427 \text{ g}$

$$\begin{cases} \dot{\omega}_x = \omega_y \Omega & \ddot{\omega}_x = \omega_y \dot{\Omega} = -\omega_x \Omega^2 \\ \dot{\omega}_y = -\omega_x \Omega & \ddot{\omega}_y = -\omega_x \dot{\Omega} = -\omega_y \Omega^2 \end{cases}$$

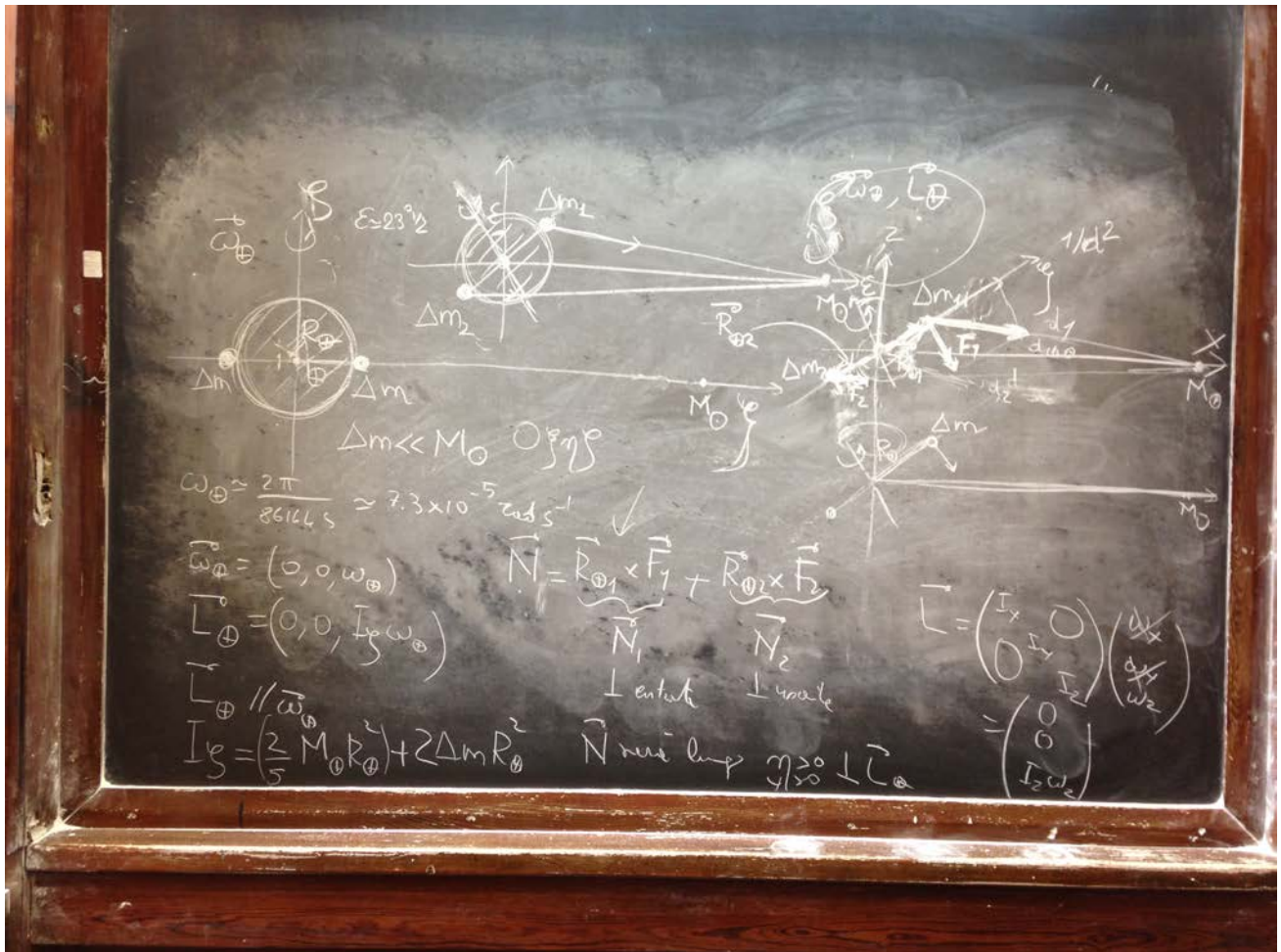
$$\begin{cases} \dot{\omega}_x + \Omega^2 \omega_x = 0 \\ \dot{\omega}_y - \Omega^2 \omega_y = 0 \end{cases}$$

22 Maggio 2014 (3 ore)

Moto del corpo rigido in presenza di momenti di forza non nulli. Il caso della precessione dell'asse di rotazione della Terra causata dalla Luna e dal Sole.

Considerazioni sulla non uniformità della forza di attrazione gravitazionale.

Altri esercizi sul calcolo del momento di inerzia



$\frac{dL}{dt} = \vec{N}$
 $\frac{dL}{dt} = \vec{N} + \vec{L} \times \vec{\omega}$
 $L \times \omega = N$
 $L \times \omega = \Omega = N$
 $\Omega = \frac{N}{L \times \omega} = \frac{N}{I \omega \times \epsilon}$
 $F_1 \approx \frac{GM_0 \Delta m}{d_1^2}$
 $F_2 \approx \frac{GM_0 \Delta m}{d_2^2}$
 $N_1 = \frac{GM_0 \Delta m R_0}{d_1^3}$
 $N_2 = \frac{GM_0 \Delta m R_0}{d_2^3}$

$(1-\epsilon)^{-2} \approx 1+2\epsilon$ K_0
 $(1+\epsilon)^{-2} \approx 1-2\epsilon$

$d_1 = d_{00} + R_0$
 $d_2 = d_{00} - R_0$

$\frac{1}{d_1^2} = \frac{1}{(d_{00} + R_0)^2} = \frac{1}{d_{00}^2 (1 + \frac{R_0}{d_{00}})^2} = \frac{1}{d_{00}^2} \left(1 - \frac{2R_0}{d_{00}}\right)$
 $\frac{1}{d_2^2} = \frac{1}{(d_{00} - R_0)^2} = \frac{1}{d_{00}^2 (1 - \frac{R_0}{d_{00}})^2} = \frac{1}{d_{00}^2} \left(1 + \frac{2R_0}{d_{00}}\right)$

$N \propto \frac{GM_0 \Delta m}{d^2} \left(\frac{R_0}{d}\right)$
 $N_0 \propto \frac{GM_0}{d_{00}^3} R_0 \Delta m$
 $N_1 \propto \frac{GM_0}{d_{00}^3} R_0 \Delta m$
 $N_2 \propto \frac{GM_0}{d_{00}^3} R_0 \Delta m$

$\frac{N_1 - N_2}{N_0} = \frac{M_1}{M_0} \frac{d_{00}^3}{d_{01}^3} \approx 2\epsilon$

$\vec{L} = I\vec{\omega}$
 $\frac{d\vec{L}}{dt} = \vec{N}$
 $\vec{L} = N$
 $\vec{\omega} = \frac{d\theta}{dt} \hat{z}$
 $L_z = I\omega_z = N$
 $\vec{L} = N \hat{z}$
 $\vec{\omega} = \frac{N}{I} \hat{z}$
 $T = \frac{2\pi}{\omega} = \frac{2\pi I}{N}$
 $F_1 \approx \frac{GM_0 \Delta m}{r^2}$
 $F_2 \approx \frac{GM_0 \Delta m}{r^2}$
 $N_1 = \frac{GM_0 \Delta m R_0}{d^2}$
 $N_2 = \frac{GM_0 \Delta m R_0}{d^2}$
 $\vec{L} = (0, 0, \frac{N}{I\omega_0 + \epsilon})$

$I_s = \frac{1}{2} MR^2$



$I_a = \frac{2}{4} \pi R^2 \rho \int_0^{h/2} dz = \frac{1}{2} \pi R^2 \rho \frac{h}{2} = \frac{1}{4} \pi R^2 \frac{M}{\pi R^2 h} \frac{h}{2}$
 $= \frac{1}{4} MR^2 + \frac{1}{12} Mh^2$
 $dm = \pi R^2 dz \rho$
 $I_b = \frac{1}{4} dm R^2 = \frac{1}{4} R^2 \pi R^2 \rho dz = \frac{1}{4} \pi R^4 \rho dz$

$I_s = \frac{1}{2} MR^2$
 $I_a = \frac{1}{4} MR^2$

$z = Ae^{i\delta}$
 $\bar{z} = A_0 e^{-i\delta}$
 $z = x + iy$
 $\bar{z} = x - iy$
 $z\bar{z} = A_0^2 e^{-i\delta} A_0 e^{i\delta} = A_0^2$

$I_x = 2 \int_0^{h/2} dm z^2 - 2z \int_0^{h/2} dm z = 2 \int_0^{h/2} \frac{z^3}{3} dz - 2z \left[\frac{z^2}{2} \right]_0^{h/2} = \frac{2}{3} \left[\frac{z^3}{3} \right]_0^{h/2} - \frac{2}{2} \left[\frac{z^2}{2} \right]_0^{h/2} = \frac{2}{9} \frac{h^3}{3} - \frac{1}{2} \frac{h^2}{2} = \frac{1}{12} \frac{M}{h} h^3 = \frac{1}{12} Mh^2$

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